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TURBULENCE IN ORTHOKINETIC FLOCCULATION

Yerachmiel Argaman, et al

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Berkeley, California

July 1968

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# **TURBULENCE IN ORTHOKINETIC FLOCCULATION**

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UNIVERSITY OF CALIFORNIA  
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TURBULENCE IN ORTHOKINETIC FLOCCULATION

by

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and  
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Sanitary Engineering Research Laboratory  
College of Engineering  
and  
School of Public Health  
University of California  
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## I. INTRODUCTION

### FLOCCULATION IN WATER AND WASTE WATER PURIFICATION

For almost a century the water purification industry has been employing the flocculation process as a preparatory step for the removal of suspended impurities from water. Except for disinfection, flocculation still remains the most used and most important method of water treatment, both in terms of volume of water treated and of chemicals used. Although a modern water treatment plant is an integrated system of several unit processes, many in the field believe that as a single process, flocculation is by far the most important. Flocculation was originally considered merely as a preparatory step to subsequent separation operations, particularly sedimentation and filtration. This was due on one hand to a lack of understanding of the chemical phenomena of coagulation and filtration and on the other to the limited task assigned to flocculators.

During the early years of water treatment bacteria, turbidity and organic color were the only impurities that had to be removed. The rapid urban and industrial developments which started in the second half of the last century have created a steady increase in water consumption and the pollution of existing sources. The water supply industry was faced with the need to provide larger quantities of water and at the same time to develop the techniques for maintaining their quality. The problem was intensified by the detection of a large variety of new substances in the raw waters. One had to add to the list of substances to be removed from water such new and exotic materials as surfactants, herbicides, pesticides, and a large number of organic and inorganic pollutants from industrial wastes. These problems have led to an increased interest in coagulation-flocculation, its basic mechanisms and purification capabilities. Studies of the chemistry of coagulation have provided considerable basic information which contributed to a better understanding of the process. This understanding is nevertheless far from complete and its complexity still provides a major challenge. One of the most dramatic developments of coagulation in the recent years is the use of a completely new type of coagulant, the polyelectrolytes. Their introduction marks the beginning of a new era in coagulation-flocculation. Used as sole coagulants or as coagulant aids, these chemicals not only increase the removal efficiency of color and turbidity, but may also be capable of removing some of the new pollutants not affected by conventional coagulants. Though flocculation is currently practiced almost exclusively in water treatment, its application for waste treatment is becoming increasingly attractive as, for example, an adjunct to phosphate precipitation. The recent trend in pollution control is from the traditional concern for biodegradable carbon and suspended solids removal to problems of nutrient and refractory removal and this has created greater interest in chemical waste treatment. In any chemical treatment of wastes, flocculation is likely to play a key

role, particularly with the availability of a large variety of new coagulants and the potential for their most efficient through rationally engineered flocculators.

#### SOME CHEMICAL ASPECTS OF COAGULATION AND FLOCCULATION

The removal of colloidal material from water in the flocculation process occurs in two phases. The first of these, usually referred to as coagulation, begins immediately after the addition of the coagulant and consists of a series of chemical reactions and physical interactions between the coagulant, the surface of the colloidal particles and the water. In the second step, destabilized particles or coagules collide with each other and form small aggregates which grow further into relatively large aggregates commonly known as flocs. The mechanism and kinetics of the second step is the subject of this report. Nevertheless, a brief review of the current views on the first step, that of coagulation, seems appropriate. The stability or resistance to flocculation of a colloidal suspension is attributed to electrostatic repulsive forces created by the electrical charges on the surface of the particles. The origin of these charges may be ionization of various groups at the solid side of the solid-liquid interface, isomorphic substitution or ion adsorption. The distribution of ionic species in the liquid adjacent to the solid surface is affected by the presence of charges on the particles surface. The ionic layer attached to the solid surface and the diffused ions of opposite charge in the liquid side of the interface form the ionic double layer to which considerable attention has been given. The distribution of electrical charges within the double layer may be represented by the potential curve as shown in Figure 1 for a negatively charged particle. Most theories suggest an exponential curve with some modifications at the solid-liquid interface. The shape of the potential curve may be altered by the addition of ionic species to the electrolyte. When moving in the electrolyte the solid particle carries with it a film of liquid which extends to the surface of shear (see Figure 1). The electrical potential at the surface of shear is known as the zeta potential, and is the one which determines the mobility of the particle when placed in an external electrical field. This potential is responsible for the repulsive forces between two particles as they approach each other. One important role of a flocculant will therefore be to suppress the zeta potential and reduce to minimum these repulsive forces. Once this has been achieved the particles are free to approach each other and as their distances of separation become small enough the short-range London-Van der Waals attraction force comes into action. As shown in Figure 1, the particles, in order to collide, must possess kinetic energy in excess of the "energy barrier."

In coagulation practice polyvalent metal salts are used as coagulants, of which the aluminum and ferric salts are by far the most important. As pointed out by Stumm and Morgan [1] these metal ions are hydrolyzed in aqueous solutions and the intermediate products before precipitation are polynuclear complexes. Matijevic *et al.* [2] and later Black and Chen [3] showed that in the case of an aluminum salt the main species present at pH lower than four is a hydrated trivalent ion,  $Al^{+++}$ .

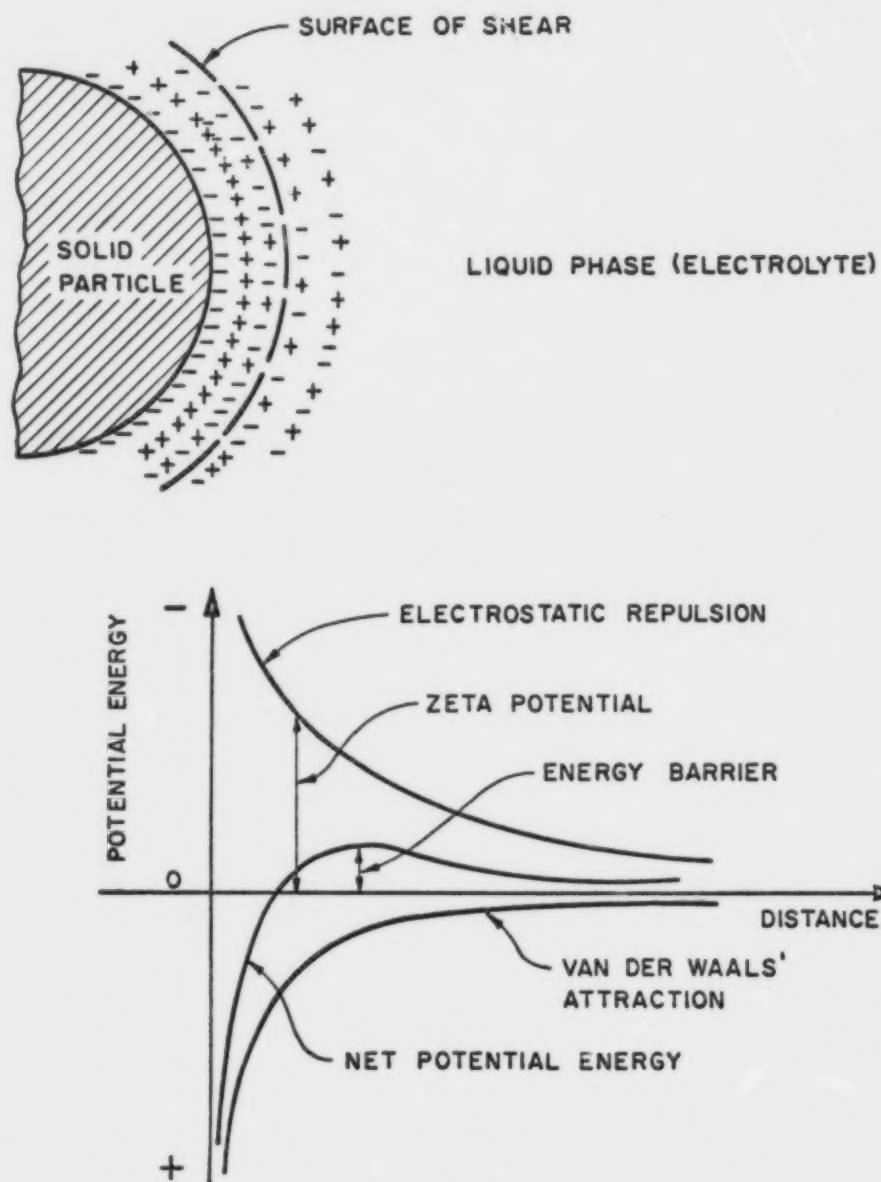


FIGURE 1. IONIC DISTRIBUTION AND POTENTIAL ENERGY AT THE SOLID-LIQUID INTERFACE

The presence of these ions in the solution results in a compression of the electrical double layer and a reduction of the zeta potential. At the pH range of four to seven polymeric species such as  $\text{Al}_3(\text{OH})_4^{+++}$  predominate. Destabilization of colloid particles at this pH range is attributed to the adsorption of these polymers onto the particles surface and reduction of the surface potential. At still higher pH or at a larger aluminum concentration most of the metal appears in the insoluble aluminum hydroxide forms. The precipitate forms relatively large micelles which tend to preferentially adsorb positive ions and thus acquire a positive charge. These micelles derive their destabilization power by either mutual coagulation with the oppositely charged colloidal particles or by providing bridging material for their agglomeration. Thus the role of the metal salt in coagulation is a combination of electrical neutralization of the negative charges on the particles surface and the formation of bridging material.

Whereas the electrical neutralization effect of a coagulant was recognized for several decades, the concept of bridging is relatively new. However, once its importance was recognized, scientific and industrial interest shifted to a new type of coagulants whose principal mechanism is probably that of bridging. These are the polyelectrolytes which are already used in some water conditioning plants as the sole coagulants and in others as coagulant aids [4]. The ionic polyelectrolytes are synthetic polymers which contain charged groups. The interaction of these ionic groups with the electrical charges of the colloidal particles leads to the adsorption of the polymer to the particle. As one part of the polymer molecule adsorbs to the surface of one particle, other parts of the same molecule, the "extended segments," are capable of adsorbing to a second particle thus forming a bridge between the two. The process then continues with the adsorption of more particles until a large aggregate is formed. This bridging model was studied extensively by LaMer and his co-workers [5] who provided a mathematical formulation for the kinetics of coagulation with polyelectrolytes.

More recent studies [6,7] have shown that the effectiveness of polymers in flocculating colloidal suspension depends on the electrical properties of both the polymers and the solid particles. Negatively charged particles, for example, are better coagulated with a cationic polymer. Changes in the electrical double layer which can be achieved by the addition of proper electrolytes can also affect the efficiency of the process. Mutual coagulation of polymers and solid particles of the same charge has also been demonstrated. Thus the mechanism of flocculation with polyelectrolytes is a combination of electrical neutralization, bridging, and mutual coagulation.

#### PREVIOUS STUDIES OF FLOCCULATION KINETICS

Interest in flocculation kinetics stems from practical considerations in the design and operation of water clarification systems. Reports on flocculation kinetics studies are scattered throughout the sanitary and chemical engineering literature of the past fifty years.



The volume of original work and the overall progress achieved during that period are not overly impressive considering the practical importance of the process. Most theoretical work has been centered on determining the frequency of collisions between suspended particles in highly idealized systems with limited attention given to the aggregates encountered in natural water clarification or the complex turbulence regimens in the stirred tank reactor.

The early theories of flocculation kinetics were presented by von Smoluchowski [8,9] who derived the basic expressions for the collision frequencies under two different collision mechanisms. For Brownian motion flocculation (perikinetic) the frequency of collisions was obtained by calculating the diffusional flux of particles towards a single stationary particle. The calculation required solution of the continuity equation from which a concentration gradient around the stationary particle was obtained. The resulting relationship for the collision frequency is given by Equation 1.

$$I_{ij} = 4\pi D_{ij} R_{ij} n_i n_j \quad (1)$$

where  $I_{ij}$  is the number of contacts made per unit time and unit volume between particles of radius  $R_i$  and those of radius  $R_j$ ;  $D_{ij}$ , the combined diffusion coefficient,  $D_i + D_j$ ;  $R_{ij}$  the collision radius which equals  $R_i + R_j$ ; and  $n_i$  and  $n_j$ , the number concentrations of particles of radius  $R_i$  and  $R_j$ .

In addition to Brownian motion collisions, which can take place with the fluid macroscopically at rest, Smoluchowski realized that suspended particles may collide with each other due to their motion with the fluid, i.e., orthokinetic flocculation.\* Under laminar flow conditions he was able to show that the collision frequency was given by

$$J_{ij} = \frac{4}{3} n_i n_j R_{ij}^3 \frac{dU}{dz} \quad (2)$$

where  $n_i$ ,  $n_j$ , and  $R_{ij}$  carry the same connotation as in Equation 1,  $dU/dz$  is the laminar velocity gradient, and  $J_{ij}$  is the collision frequency due to laminar fluid motion.

The next major development in the theory of flocculation kinetics was provided by Camp and Stein [10], who observed that in most practical cases flocculation was achieved under turbulent flow conditions and attempted to generalize Smoluchowski's equation to

---

\* Smoluchowski is believed to have selected the term "ortho" to describe the ordered nature of shear flocculation in contrast to the disordered nature of Brownian or perikinetic flocculation. In this report ortho refers to any form of flocculation caused by fluid motion.

include these conditions. Adopting Smoluchowski's model for particle's collisions by fluid motion, they were still faced with the problem of identifying the turbulent equivalent of  $dU/dz$ . The problem originates from the basic difference between laminar and turbulent flows. In the latter case, even at steady state, the fluid velocity fluctuates around a temporal mean value, creating local microscopic velocity gradients superimposed on the macroscopic gradients of the mean velocity. To overcome this difficulty Camp and Stein have adopted a formal procedure by which they were able to substitute a measurable parameter for  $dU/dz$ . They defined an "absolute velocity gradient at a point,"  $G_p$  which is given by

$$G_p = \left[ \left( \frac{dU}{dy} + \frac{dV}{dx} \right)^2 + \left( \frac{dU}{dz} + \frac{dW}{dx} \right)^2 + \left( \frac{dV}{dz} + \frac{dW}{dy} \right)^2 \right]^{\frac{1}{2}} \quad (3)$$

where U, V, and W are the three velocity components in the x, y, and z directions respectively. According to Stokes [11] this velocity gradient equals the square root of the energy dissipation at a point divided by the fluid kinematic viscosity. Camp and Stein have used Stokes' theory to relate the total energy input into the fluid to what they called a root-mean-square velocity gradient. Thus, according to their generalization,

$$G = \sqrt{\frac{\epsilon}{\nu}} \quad (4)$$

where  $\epsilon$  is the total energy dissipated per unit time and unit fluid mass, and  $\nu$  is the kinematic viscosity. Hence, according to Camp and Stein, the rate of collisions between particles suspended in a turbulent fluid is given by

$$H_{ij} = \frac{4}{3} n_i n_j R_{ij}^3 G \quad (5)$$

This equation has generally been accepted by practicing sanitary engineers and has served as a basis for the development of design criteria for flocculation reactors.

The principal objection to Equation 5 stems from the fact that the overall root-mean-square velocity gradient does not take into account the length scale over which these local and temporary velocity gradients extend. Velocity gradients of a given length scale will not contribute much to the collision rate of particles which are larger than this scale. In order to use Smoluchowski's procedure for calculating collision frequencies a detailed picture of the velocity gradients distribution in both time and space is required. The substitution of  $G$  for  $dU/dz$  in Equation 2 is therefore more of an arbitrary procedure, than it is a basic physical truth. For this reason a search for better and more rational theories for turbulent flocculation has continued. One approach was proposed by Frisch [12]. Assuming that the motion of



the suspended particles characterizes the motion of the fluid medium itself, he was able to arrive at an expression for the collision rate between suspended particles based on a diffusion model similar to Smulochowski's treatment of Brownian motion flocculation. The theory was limited, however, to particles which were very small compared with the microscale of the turbulence. A diffusion model was also adopted by Levich [13] who derived the following expression for the collision frequency of very small particles in a turbulent field:

$$H_{ij} = 12\pi\beta\sqrt{\frac{\epsilon}{\nu}} n_i n_j R_{ij}^3 \quad (6)$$

where  $\beta$  is a constant.

Except for the numerical constant, this equation is equivalent to Equation 5. It has however a more rational basis as it was developed from a pure mechanistic model. In his derivation, Levich assumed that the diffusional motion of the suspended particles which was responsible for the interparticle collisions was affected by turbulent eddies similar in size to the collision radius. By further assuming that this radius was smaller than the microscale of the turbulence, he was able to arrive at a relatively simple expression for the collision frequency.

The rate of collisions between liquid particles suspended in a turbulent fluid was studied by Saffman and Turner [14] with regard to the formation of rain drops. For nearly equal drops the collision frequency was found to be

$$H_{ij} = 1.30 R_{ij}^3 n_i n_j \sqrt{\frac{\epsilon}{\nu}} \quad (7)$$

which is almost identical to the expression derived independently by Levich. The validity of Equation 7 is limited to cases where the particles are smaller than the microscale of the turbulence by at least one order of magnitude.

Howarth [15] has pointed out that a lasting collision will occur only when the colliding particles approach each other with sufficient kinetic energy to squeeze out the liquid layer between them. To account for this, he suggested that a probability factor be included in the rate equations and that it be proportional to the fraction of particles having kinetic energies larger than some predetermined level assuming Maxwellian velocity distribution. Once a collision mechanism is assumed, and an expression for the collision frequency is selected, a complete kinetic equation for the flocculation process can be derived. For orthokinetic flocculation, i.e., flocculation which is brought about by fluid motion, either laminar or turbulent, using Smoluchowski's approach, Fair and Gemmell [16] have presented the flocculation kinetic equation in the form

$$\frac{dn_k}{dt} = \frac{2}{3} \left[ \sum_{i=1}^{k-1} \sum_{j=k-i}^{k-1} n_i n_j R_{ij}^3 - 2 n_k \sum_{i=1}^{\infty} n_i R_{ik}^3 \right] \frac{dU}{dz} \quad (8)$$

This equation describes the rate of change in the number concentration of particles of a given radius,  $R_k$ . The first term inside the brackets depicts the formation of  $k$ -fold particles from collisions between  $i$ -fold and  $j$ -fold particles where  $i + j = k$ . The second term represents the elimination of  $k$ -fold particles due to their collisions with all other particles. The assumption was made that all collisions were lasting, i.e., result in aggregation. Floc breakup was not included.

The solution of Equation 8 with the appropriate boundary conditions for different collision and breakup mechanisms has been the subject of several studies in recent years. Swift and Friedlander [17] and later Chiu-Sen Wang [18] have solved the equation replacing the summations by integrals, and obtained analytical solutions for several collision mechanisms in batch systems. They have shown that for Brownian motion and shear flow coagulation, the floc size distribution function is independent of the initial particle size distribution and approaches a self-preserving form which depends on the collision mechanism alone.

Equation 8 assumes a considerably simpler form when written for  $k = 1$  and this particular case is of great practical significance as it describes the rate at which 1-fold (primary) particles are removed from the system. Harris and Kaufman [19] and Harris, Kaufman and Krone [20] have modified Equation 8 to accommodate for floc breakup and for possible nonlasting collisions. They derived the following expression for the rate of change in primary particle concentrations:

$$\frac{dn_1}{dt} = - \frac{\alpha a^3}{\pi} \delta \phi n_1 \frac{dU}{dz} \quad (9)$$

where  $\alpha$  is the fraction of collisions which result in aggregation,  $a$  is the ratio of the collision radius of a floc to its physical radius,  $\phi$  is the floc volume fraction and  $\delta$  is a size distribution function defined by

$$\delta = \frac{\sum_{i=0}^{p-1} n_i [i^{\frac{1}{3}} + 1]^3}{\sum_{i=0}^p i n_i} \quad (10)$$

The maximum floc size permitted was p-fold. Several other distribution functions were obtained for various floc formations and breakup models.

A somewhat different approach was taken by Hudson [21] who argued that in a continuous flow system a simplified bimodal floc size distribution can be assumed. If the system is composed only of primary particles and large flocs and if the size variations within each group are small compared to the difference between the groups, then the removal of primary particles will occur primarily by their collisions with flocs. Under these conditions Equation 8 may be simplified to read

$$\frac{dn_1}{dt} = -\frac{4}{3} n_1 n_F R_F^3 \frac{dU}{dz} \quad (11)$$

where  $R_F$  is the radius of flocs, and  $n_F$  is their number concentration. As the product  $n_F R_F^3$  is proportional to the total floc volume, Hudson concluded that the floc volume fraction was of major importance.

#### OBJECTIVES OF THE CURRENT INVESTIGATION

The general objective of this study was to show the relationship between the rate of orthokinetic flocculation and the hydrodynamic parameters describing the internal structure of turbulence in stirred reactors. Specifically, the study had the following objectives:

1. Derivation of a theoretical expression for the frequency of collisions between particles suspended in a turbulent fluid, in which the particles are larger than the smallest scale of the turbulence.
2. Development of a working formula expressing the degree of flocculation obtained in a continuous flow system as a function of the hydrodynamic parameters and physical properties of the system.
3. Laboratory verification of proposed theory through the simultaneous determination of flocculation performance and hydrodynamic characteristics.
4. Development of design criteria and operational considerations for full scale treatment plants.

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## II. PARTICLE COLLISION IN A TURBULENT FLUID

### INTRODUCTION

The essence of the flocculation process is the aggregation of suspended coagulated particles to form larger flocs amenable to separation from the suspending medium by some subsequent physical process, generally sedimentation. This cannot proceed at an appreciable rate unless proper conditions prevail which make the separated particles approach each other close enough for the attraction forces to overcome those of repulsion and produce a lasting aggregation. The rate at which aggregates form is dependent on the collision rate. This is true regardless of whether all collisions result in aggregation or only a fraction remain aggregated. Thus, a rational study of the factors affecting the rate of flocculation should be based on the rate mechanism of particle collisions. This rate is in turn based on the motion of the individual particles, and particularly on their relative motion towards each other.

In this chapter a theoretical approach to the collision mechanism between suspended particles in a turbulent fluid is presented. It has been recognized that the proposed model should be based on rational physical arguments while at the same time lending itself to quantitative treatment. The basic assumption was that the principal collision mechanism was one in which the particles are brought to contact with each other through motion with their surrounding fluid. This assumption is justified in the present case as the density of the suspended particles is very close to that of the fluid. Thus the problem reduces to investigating the details of the fluid motion. Specifically it centers on the relative motion of adjacent fluid masses.

One possible approach to this problem is through the analysis of velocity gradients. In principle a collision rate equation can be derived if the velocity gradients are known in detail. This would require information regarding the space and time behavior of the velocity field at each point in the fluid medium. This information is required because two suspended particles will collide due to a certain velocity gradient only if they are both within the fluid volume in which this gradient occurs, and if the duration of the gradient is long enough to permit collision. Apart from the complexity of formulating the collision frequency by such a model, there is the very difficult problem of obtaining the required information regarding the velocity gradients. Because of the formidable aspects of the velocity gradient approach an alternative model was adopted.

In Brownian motion flocculation, Smoluchowski was able to calculate the collision rate by using a diffusion model. An analogous procedure is proposed here. While making this analogy it is recognized that some basic differences between the fine structures of turbulent flow and Brownian motion exist and should be taken into consideration.



Diffusion theory is based on the random walk concept which implies that the motion of a particle at any given time is independent of its previous motions. In turbulent flow, this condition is only partially satisfied; namely the motion of a particle does depend on its history but only on the very recent portion of it. This property of turbulent flow is expressed by the time correlation function.

The proposed theory centers on the hypothesis that particles suspended in a turbulent fluid experience a random motion resembling gas molecules and that this motion can be characterized by an appropriate diffusion coefficient. The subject of this chapter is the development of a rational expression for this diffusion coefficient in terms of the measurable hydrodynamic properties of the fluid. As the suspended particles are carried by their surrounding fluid, the theory starts with the expression for the turbulent diffusion of fluid particles. Two alternative ways for describing the turbulence field exist from which an expression for the diffusion coefficient can be deduced. One involves the concept of eddy motions while the other centers on the turbulence energy spectrum.

In the eddy approach to turbulence an eddy is defined as a group of neighboring fluid particles which at a given time move at the same velocity, or almost the same velocity. An expression analogous to molecular diffusion for the turbulent diffusion coefficient can be obtained by a product of eddy size and eddy velocity. Suspended particles will be completely entrained by eddies which are larger than the particle size and thus their diffusivity will be affected mainly by the motion of these eddies. On the other hand, the relative motion of two particles will be primarily affected by eddies which are equal or smaller than their separation. Hence it is evident that diffusional motion of two particles towards each other will be governed by eddies which are larger than their size and smaller than their separation. As the description of turbulence by eddy motions does not lend itself to a quantitative mathematical treatment, an alternative approach is adopted. Nevertheless the above arguments provide a physical picture of the problem.

The alternative approach, one based on describing the turbulence field in terms of its energy spectrum, is presented in the following sections. This model leads to a convenient working equation for flocculation kinetics which proved to describe adequately the experimental results. In the theoretical derivations only those aspects of turbulent flow which are intimately associated with the subsequent derivations are discussed, and no attempt is made to present an exhaustive review of the numerous publications on turbulence.

#### CORRELATION AND THE ENERGY SPECTRUM IN TURBULENCE

At a fixed point in a steady turbulent field the velocity of the fluid in a certain direction, e.g., along the  $x$  axis may be given by the equation



$$U = \bar{U} + u \quad (12)$$

where  $U$  is the instantaneous velocity,  $\bar{U}$  is the temporal mean velocity, and  $u$  is the instantaneous deviation from the temporal mean or the fluctuating velocity. The relationship among these three variables is shown in Figure 2. By definition the average of  $u$  is zero. However, the average of  $u^2$  is a finite constant if the turbulence field is steady and is statistically homogeneous with respect to time. A common and powerful tool for studying the properties of a time function like  $u$  is the Fourier analysis. If it is assumed that the fluctuating velocity can be resolved into a series of harmonic components,  $u$  can be expressed as a Fourier integral,

$$u(t) = 2\pi \int_0^{\infty} [a(f) \cos 2\pi ft + b(f) \sin 2\pi ft] df \quad (13)$$

where  $f$  is the frequency, and the coefficients  $a(f)$  and  $b(f)$  are defined by

$$a(f) = \frac{1}{\pi} \int_{-\infty}^{\infty} u(t) \cos 2\pi ft dt \quad (14)$$

and

$$b(f) = \frac{1}{\pi} \int_{-\infty}^{\infty} u(t) \sin 2\pi ft dt. \quad (15)$$

We now introduce the Eulerian time correlation function which is defined as the average value of the product  $u(\tau) u(\tau-t)$  at a fixed point. The Eulerian time correlation coefficient is given by

$$R_E(t) = \frac{\overline{u(\tau) u(\tau-t)}}{\overline{u^2}} \quad (16)$$

where  $\overline{u^2}$  is the mean square fluctuating velocity. If the turbulence is observed over a time interval of  $2T$ , it can be shown [22], using Equation 13 for  $u(t)$ , that

$$\overline{u(\tau) u(\tau-t)} = \frac{4\pi^2}{2T-t} \int_0^{\infty} \frac{a^2(f) + b^2(f)}{2} \cos 2\pi ft df. \quad (17)$$

For  $t = 0$  the correlation function reduces to the mean square velocity fluctuation which according to Equation 17 is given by

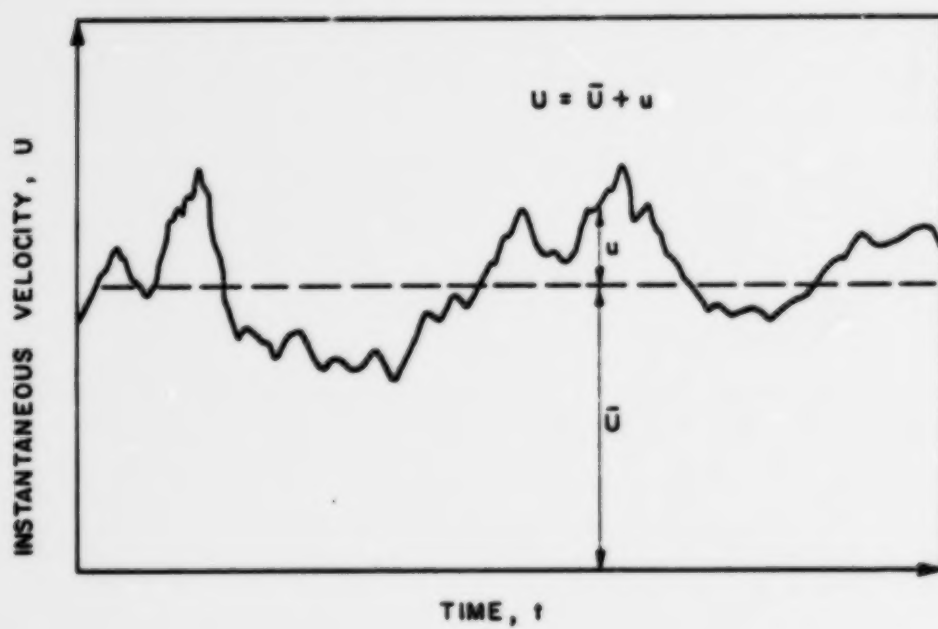


FIGURE 2. VELOCITIES IN TURBULENT FLOW

$$\overline{u^2} = \frac{\pi^2}{T} \int_0^{\infty} [a^2(f) + b^2(f)] df \quad (18)$$

or

$$\overline{u^2} = \int_0^{\infty} E_1(f) df \quad (19)$$

where

$$E_1(f) = \frac{\pi^2}{T} [a^2(f) + b^2(f)] . \quad (20)$$

$E_1(f)$  is referred to as the "unidimensional energy spectrum function." It represents the contribution to  $\overline{u^2}$  from motions of frequencies between  $f$  and  $f + df$ .

From Equations 16, 17 and 20 with  $T \gg t$  the correlation coefficient can be written as

$$R_E(t) = \frac{1}{\overline{u^2}} \int_0^{\infty} E_1(f) \cos 2\pi f t df . \quad (21)$$

If both sides of this equation are multiplied by  $\cos 2\pi f t$  and integrated with respect to  $t$  from  $-T$  to  $+T$  the result is

$$E_1(f) = 4\overline{u^2} \int_0^{\infty} R_E(t) \cos 2\pi f t dt . \quad (22)$$

Hence it is seen from Equations 21 and 22 that the unidimensional energy spectrum function,  $E_1(f)$ , and the correlation function,  $\overline{u^2} R_E(t)$ , are Fourier cosine transforms of each other. This fact, which was first pointed out by Taylor [23], is of great practical significance in this study as it enables the determination of the energy spectrum from direct measurement of the correlation.

#### TURBULENT DIFFUSION

Experience shows that in a fully developed turbulent environment the motions of fluid particles are randomly distributed very much like the motions of gas molecules. An analytical basis for this observation is given by Roberts [24]. If a number of neighboring particles is considered at one instant and their position at subsequent instants is observed, a gradual spread of the particles throughout the space is noticed. This is the basic idea of diffusion. The diffusional motion

of particles is characterized by their diffusion coefficient which is a measure of the spreading tendency. In the kinetic theory of gases the diffusion coefficient is given by the product of the molecular velocity and the mean free path. An analogous expression for turbulent fluids would be the product of eddy velocity and eddy scale or wavelength. However, in the case of turbulence an entire spectrum of eddy scales and velocities exist, although a particular diffusion-dependent process (e.g., flocculation) may be influenced principally by eddies of certain sizes.

The description of a turbulent field by eddy motions does not lend itself to a quantitative mathematical treatment. On the other hand, the spectrum picture is more difficult to visualize. Because the objective of this research is to arrive at a quantitative relationship between the kinetics of flocculation and the hydrodynamic properties of the turbulence, the more quantitative approach to turbulence, namely the energy spectrum, has been adopted. It should be recognized, however, that the eddy picture and the spectrum analysis are closely related to each other.

The analytical basis for characterizing the diffusional properties of a turbulent field are presented in this section. Most derivations are due to Taylor [23]. If  $v(t)$  is the velocity of a fluid particle in the  $y$  direction at time  $t$ , the position of a particle initially at  $y(t_0)$  is given by

$$y(t_0 + t) = y(t_0) + \int_0^t v(t_0 + t') dt' . \quad (23)$$

If many particles, all starting at the same point but at different times, are observed under stationary conditions, the average displacement,  $y(t_0 + t) - y(t_0)$ , will be zero. This is because the turbulence motion is completely random. The average square of the displacements during a time  $T$  will usually differ from zero. It is obtained by

$$\overline{y^2}(t + t_0) = \frac{1}{T} \int_0^T y^2(t + t_0) dt_0 . \quad (24)$$

Selecting  $y(t_0) = 0$  and substituting Equation 23 into 24 yields

$$\overline{y^2}(t + t_0) = \frac{1}{T} \int_0^T dt_0 \int_0^t v(t_0 + t') dt' \int_0^t v(t_0 + t'') dt'' . \quad (25)$$

The integration with respect to  $t_0$  from 0 to  $T$  and division by  $T$  is equivalent to the averaging operation. Hence,

$$\overline{y^2}(t_0 + t) = \int_0^t v(t_0 + t') dt' \int_0^t v(t_0 + t'') dt'' . \quad (26)$$

Selecting  $t_0 = 0$  and noting that the average of a summation is equal to the summation of the averages, Equation 26 reduces to

$$\overline{y^2}(t) = \int_0^t dt' \int_0^t dt'' \overline{v(t') v(t'')} . \quad (27)$$

The integration in the  $(t'', t')$  plane over a square with limits 0 to  $t$  can be replaced by twice the integration over a triangle in the following way

$$\int_0^t dt' \int_0^t dt'' = 2 \int_0^t dt' \int_0^{t'} dt'' . \quad (28)$$

Thus Equation 27 can be transformed to

$$\overline{y^2}(t) = 2 \int_0^t dt' \int_0^{t'} \overline{v(t') v(t'')} dt'' . \quad (29)$$

Defining a time lag  $\tau = t' - t''$ , the equation further reduces to

$$\begin{aligned} \overline{y^2}(t) &= -2 \int_0^t dt' \int_{t'}^0 \overline{v(t') v(t' - \tau)} d\tau \\ &= 2 \int_0^t dt' \int_0^{t'} \overline{v(t') v(t' - \tau)} d\tau . \end{aligned} \quad (30)$$

The average of the product  $v(t') v(t' - \tau)$  is the Lagrangian correlation function which is related to the Lagrangian correlation coefficient by

$$R_L(\tau) = \frac{\overline{v(t) v(t - \tau)}}{\overline{v^2}} . \quad (31)$$

In terms of the Lagrangian correlation coefficient, the mean square of the displacement becomes



$$\overline{y^2}(t) = 2 \overline{v^2} \int_0^t dt' \int_0^{t'} R_L(\tau) d\tau . \quad (32)$$

By carrying out a partial integration on Equation 32, it can be altered to

$$\overline{y^2}(t) = 2 \overline{v^2} \int_0^t (t - \tau) R_L(\tau) d\tau . \quad (33)$$

The relation between the Lagrangian correlation function and the Lagrangian energy spectrum,  $E_L(f)$ , is analogous to that in Eulerian terms. Hence, according to Equation 16,

$$R_L(\tau) = \frac{1}{\overline{v^2}} \int_0^\infty E_L(f) \cos 2\pi f \tau df . \quad (34)$$

Substitution of this expression into Equation 33 results in

$$\overline{y^2}(t) = 2 \int_0^\infty E_L(f) df \int_0^t (t - \tau) \cos 2\pi f \tau d\tau . \quad (35)$$

Performing the integration over  $\tau$ ,

$$\overline{y^2}(t) = 2 \int_0^\infty E_L(f) \frac{1 - \cos 2\pi f t}{4\pi^2 f^2} df . \quad (36)$$

The diffusion coefficient in the  $y$  direction is related to the mean square displacement by

$$D_T(t) = \frac{1}{2} \frac{d}{dt} (\overline{y^2}) . \quad (37)$$

Hence, from Equations 36 and 37,

$$D_T(t) = \int_0^\infty E_L(f) \frac{\sin 2\pi f t}{2\pi f} df . \quad (38)$$

Equation 38 provides the relation between the diffusion coefficient of fluid particles in a single direction and the Lagrangian energy spectrum function. This equation will be used in subsequent sections for deriving

the diffusion coefficient of suspended particles by assuming that the turbulence is isotropic at a single point and varies little between closely spaced points.

#### THE MOTION OF SUSPENDED PARTICLES IN TURBULENT FLUID

Discrete particles suspended in a turbulent fluid will experience a random motion with a diffusion coefficient which generally differs from that of the fluid. Relatively little theoretical work has been done on this subject. The most extensive work was done by Tchen [25] whose analysis was restricted to spherical particles which were much smaller than the smallest eddy size of the turbulence. The expression obtained for the diffusion coefficient of the suspended particles was identical to the one for the fluid, Equation 38, but the energy spectrum was related to the correlation function of the suspended particles rather than the fluid particles. Thus Tchen defines the correlation coefficient for suspended particles by

$$R_P(t) = \frac{\overline{v_P(t) v_P(t + \tau)}}{\overline{v_P^2}} \quad (39)$$

The particle energy spectrum is related to the correlation coefficient by an expression similar to Equation 34. Thus

$$R_P(t) = \frac{1}{\overline{v_P^2}} \int_0^\infty E_P(f) \cos 2\pi f t \, df \quad (40)$$

where  $E_P(f)$  is the energy spectrum of the suspension. Similarly, the diffusion coefficient of the suspended particles is given by

$$D_P(t) = \int_0^\infty E_P(f) \frac{\sin 2\pi f t}{2\pi f} \, df \quad (41)$$

The correlation and diffusion coefficients of the suspended particles differ from those of the fluid only when the velocity of the particles differs from that of their surrounding fluid. According to Friedlander [26] the relative velocity between a suspended particle and its surrounding fluid is given by

$$v_r^2 = (1 - \gamma)^2 \overline{v^2} \left[ 1 - \beta \int_0^\infty e^{-\beta t} R_L(t) \, dt \right] \quad (42)$$

where  $v_r = v_p - v$ ,  $v_p$  is the particle velocity,  $v$  is the fluid velocity,  $\gamma$  is the ratio of fluid density to particle density,  $\beta$  is a measure of particle inertia, and  $R_L(t)$  is the Lagrangian correlation coefficient for fluid velocities measured along a suspended particles path.

As in the current study the particles of concern have a density very close to that of the fluid, i.e.,  $\gamma \approx 1.0$ , it will be assumed that  $v_p \approx v$ . Hence for very small particles the diffusion coefficient is equal to that of the fluid particles themselves. Particles which are larger than the small scale of the turbulence do not follow exactly the motion of their surrounding fluid and thus their diffusion coefficients differ from that of the fluid particles. In the extreme case where the particles are much larger than all eddies, their diffusivity approaches zero. A similar effect is known in Brownian motion where the diffusivity of suspended particles rapidly decreases as their sizes become much larger than the molecular mean free path. In order to facilitate the subsequent theoretical derivations, the diffusion coefficient of the particles of any size will be calculated from a modified version of Equation 41. An "entrainment factor" will be introduced to account for the fact that suspended particles are only partially entrained by eddies smaller than their diameters. The entrainment factor selected here is of a very simple form and has no theoretical justification other than being intuitively reasonable. It is defined by

$$e_f = \frac{\lambda}{2R + \lambda} \quad (43)$$

where  $\lambda$  is the eddy scale and  $R$  is the particle radius. With the entrainment factor, Equation 41 can be used for calculating the diffusion coefficient of particles of any size with  $E_L(f) \cdot e_f$  replacing  $E_P(f)$ . Thus the diffusion coefficient for particles of any size is estimated by

$$D_P(t) = \int_0^{\infty} E_L(f) \frac{\sin 2\pi f t}{2\pi f} \frac{\lambda}{2R + \lambda} df \quad (44)$$

To express  $D_P(t)$  in terms of a single variable, say the frequency, an expression relating the scale  $\lambda$  to the frequency  $f$  should be used. Levich [13] gives the following relation for eddies larger than  $\lambda_0$ ,  $\lambda_0$  being the size of an eddy for which the Reynolds number is unity.

$$\frac{1}{2\pi f} = \left( \frac{1}{\epsilon} \right)^{\frac{1}{3}} \lambda^{\frac{2}{3}} \quad (45)$$

From this equation  $\lambda$  can be expressed as a function of  $f$ . Thus

$$\lambda = \left( \frac{1}{2\pi} \right)^{\frac{3}{2}} e^{\frac{1}{2}} r^{-\frac{3}{2}} \quad (46)$$

or

$$\lambda = A r^{-\frac{3}{2}} \quad (47)$$

where

$$A = \left( \frac{1}{2\pi} \right)^{\frac{3}{2}} e^{\frac{1}{2}} .$$

Substituting for  $\lambda$  in Equation 44,

$$D_P(t) = \int_0^{\infty} E_L(r) \frac{\sin 2\pi f t}{2\pi f} \frac{A r^{-\frac{3}{2}}}{2R + A r^{-\frac{3}{2}}} dr \quad (48)$$

This expression gives the spectral contribution to the diffusion coefficient of particles of radius  $R$ . It will be used in subsequent derivations for calculating the collision frequency between suspended particles.

#### RATE OF COLLISIONS BETWEEN DIFFUSING PARTICLES

The collision frequency between diffusing particles can be determined by calculating the rate at which particles approach a single stationary particle to within a distance equal to their collision radius. The collision radius is defined as the maximum distance of separation between the centers of two particles for which aggregation is assured. The first step in this calculation requires the solution of the particle continuity equation, which, for the case of pure diffusion, reads

$$\frac{\partial n}{\partial t} = \text{div} (D \text{ grad } n) \quad (49)$$

where  $n$  is the number concentration, and  $D$  is the diffusion coefficient. In the present case  $D$  is an overall turbulent diffusion coefficient and includes those convective motions which contribute to relative velocities between particles. Since the temporal variation in  $n$  is relatively slow, it can be assumed a constant over a period of time comparable to the time scale of the diffusion process. Thus the problem is treated as a quasi steady state. Under these conditions, Equation 49 reduces to

$$\text{div} (D \text{ grad } n) = 0 \quad (50)$$

In polar coordinates with spherical symmetry taken into account, the equation reduces to

$$\frac{\partial}{\partial r} \left( Dr^2 \frac{\partial n}{\partial r} \right) = 0 \quad . \quad (51)$$

This equation can be solved by two successive integrations. Thus

$$Dr^2 \frac{dn}{dr} = C_1 \quad (52)$$

$$n = \int \frac{C_1}{Dr^2} dr + C_2 \quad . \quad (53)$$

For a constant diffusion coefficient the integration can be completed to yield

$$n = -\frac{C_1}{Dr} + C_2 \quad . \quad (54)$$

The constants of integration  $C_1$  and  $C_2$  can be evaluated using the following boundary conditions:

$$n = n_0 \quad \text{at } r \rightarrow \infty$$

$$n = 0 \quad \text{at } r = R_{ij}, \quad R_{ij} \text{ being the collision radius.}$$

Thus,

$$C_2 = n_0$$

$$C_1 = D R_{ij} n_0$$

and Equation 54 becomes

$$n = n_0 - \frac{R_{ij} n_0}{r} \quad . \quad (55)$$

The flux of particles into a sphere of influence of radius  $R_{ij}$  is

$$\psi = 4\pi R_{ij}^2 \left[ D \frac{dn}{dr} \right]_{r=R_{ij}} \quad . \quad (56)$$



From Equation 55

$$\left[ \frac{dn}{dr} \right]_{r=R_{1j}} = \frac{n_o}{R_{1j}} \quad (57)$$

Hence the flux of particles towards the single stationary particle is

$$\psi = 4\pi R_{1j} D n_o \quad (58)$$

If the "stationary" particle is of radius  $R_i$  and concentration  $n_i$  and the diffusing particles colliding with it are of radius  $R_j$  and concentration  $n_j$  the total number of collisions between these particles is

$$H_{ij} = 4\pi R_{1j} D_{ij} n_i n_j \quad (59)$$

where  $D_{ij}$  is the mutual diffusivity of the colliding particles.

Equation 59 is equivalent to Equation 1 which was derived by Smoluchowski for Brownian motion flocculation, the only difference lies in the definition of the diffusion coefficient. In the Brownian motion case the diffusivity is given by Einstein's equation [27]

$$D = \frac{kT}{3\pi\mu a} \quad (60)$$

where  $k$  is the Boltzman constant,  $T$  is the temperature,  $a$  is the particle diameter, and  $\mu$  is the viscosity of the fluid.

In the turbulent case the diffusion coefficient is given by Equation 48. The relation between the single particle diffusion coefficient and the combined diffusion coefficient for two particles is usually taken as

$$D_{ij} = D_i + D_j \quad (61)$$

This relationship is based on the assumption that the motions of the two particles are completely independent. In the case of turbulent diffusion, this assumption cannot be made as the velocities of two particles are correlated (i.e., interdependent), at least when the particles are not too far apart. Two particles located within a single eddy do not move relative to each other due to the motion of that eddy. Thus, the largest eddies which can contribute to the relative motion of two particles are of a scale approximately equal to the interparticle separation. Similarly, the collision between diffusing particles is

governed by their relative motion when their separation is about the magnitude of the collision radius. Hence, only eddies which are smaller than the collision radius contribute to the collision rate. This qualitative argument is expressed quantitatively by setting a lower limit of integration in the expression for the turbulent diffusion coefficient (Equation 48). With this modification, an "effective diffusion coefficient" can be defined as

$$D_{\text{eff}} = \int_{f_0}^{\infty} E_L(f) \frac{\sin 2\pi f t}{2\pi f} \frac{A f^{-\frac{3}{2}}}{2R_i + A f^{-\frac{3}{2}}} df + \int_{f_0}^{\infty} E_L(f) \frac{\sin 2\pi f t}{2\pi f} \frac{A f^{-\frac{3}{2}}}{2R_j + A f^{-\frac{3}{2}}} df \quad (62)$$

by virtue of Equations 48 and 61.

The lower limit of integration,  $f_0$ , is the frequency which corresponds to eddies of the same magnitude as the collision radius. Thus, according to Equation 47

$$f_0 = A^{\frac{2}{3}} R_{ij}^{-\frac{2}{3}} \quad (63)$$

The collision frequency between suspended particles in turbulent flow can now be calculated by substituting  $D_{\text{eff}}$  for  $D_{ij}$  into Equation 59. Numerical values for the diffusion coefficient can be obtained if all terms in Equation 62 are known. Some of its qualitative properties are discussed below.

Each integrand in Equation 62 is a product of three functions: namely, the Lagrangian energy spectrum  $E_L(f)$ , the entrainment factor,  $A f^{\frac{2}{3}} / 2R + A f^{-\frac{3}{2}}$ , and a time function,  $\sin 2\pi f t / 2\pi f$ , which is sometimes referred to as the "diffusivity filter function" [28]. The entrainment factor is a simple function of the frequency and is easily determined provided the particle's radius is known. A hypothetical plot of the function is shown in Figure 3. The time factor,  $\sin 2\pi f t / 2\pi f$ , is a sine wave of time dependent period. For very short times the period is very long and the factor reduces to a constant, while for longer times the period decreases with time. Hypothetical plots of the diffusivity filter function are shown in Figure 4.

A certain difficulty arises in the determination of the Lagrangian energy spectrum. The problem stems from the fact that Lagrangian properties are not readily measurable. The formal procedure for determining the Lagrangian energy spectrum is a Fourier transformation of the Lagrangian correlation function according to Equation 22. Experimental measurement of this function requires the determination

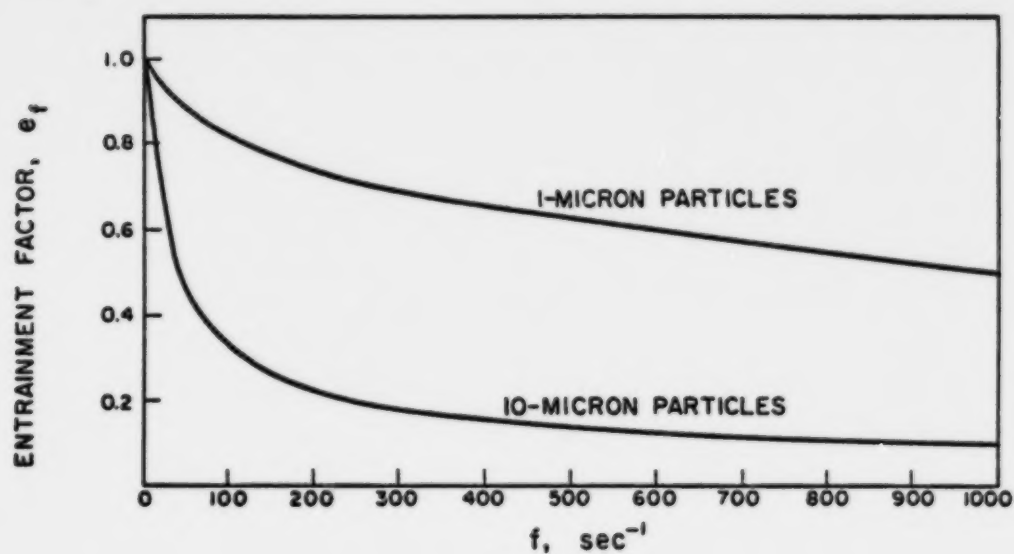


FIGURE 3. THE ENTRAINMENT FACTOR AT  $G=2.5 \text{ sec}^{-1}$

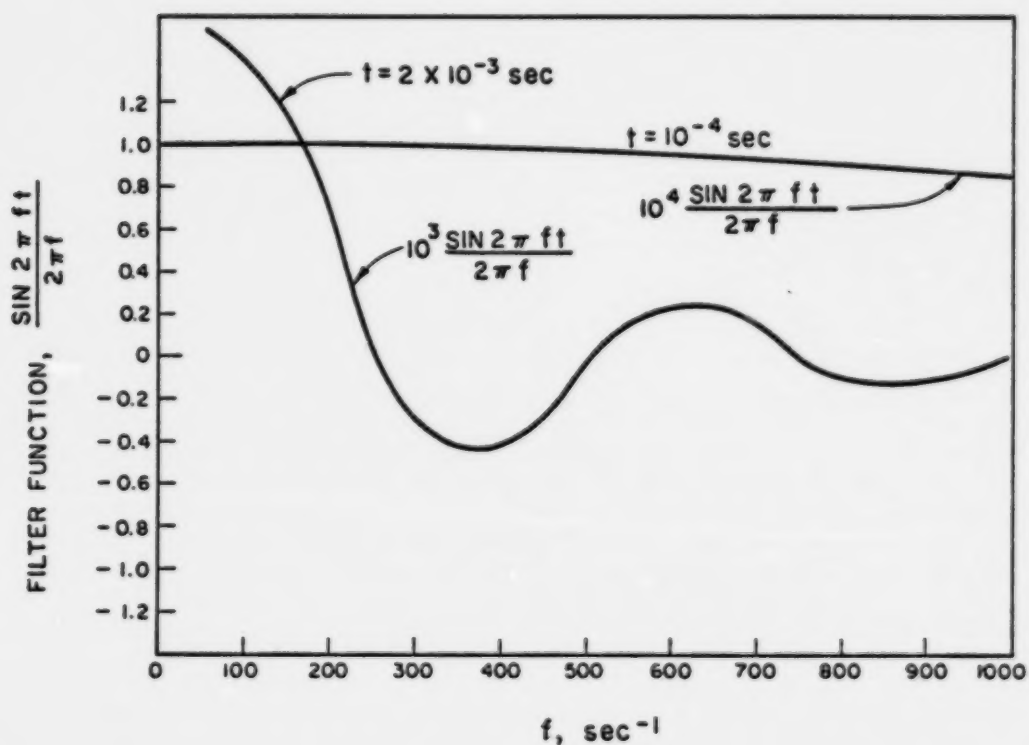


FIGURE 4. THE DIFFUSIVITY FILTER FUNCTION

of the velocity history of a single tagged fluid particle. Such a measurement should be repeated a large number of times and analyzed statistically. Obviously, this task requires highly sophisticated experimental techniques which are not within the reach of this investigation. The more readily measurable correlation and energy spectrum functions are those of the Eulerian type. Unfortunately there is no theoretical relation between the Eulerian and Lagrangian correlation functions [29,30]. In fact, this is one of the more challenging and controversial questions of present time hydrodynamics, and is beyond the scope of the current research. For the practical purposes of this study, the Eulerian type energy spectrum was measured and substituted for the Lagrangian. This arbitrary substitution has little justification other than intuition and practical necessity. Figure 5 shows a hypothetical plot of the energy spectrum, the entrainment factor, the diffusivity filter function, and the product of the three functions. The area under the latter curve from  $f_0$  to infinity is proportional to the effective diffusion coefficient. Figure 5 illustrates the interrelation that exists between the energy spectrum, the size of the colliding particles and the effective diffusion coefficient. The effect of particle size on the effective diffusivity is composed of two opposing factors; an increase in particle size results in a decrease of the entrainment factor and an increase of the collision radius. As the diffusivity is directly proportional to the entrainment factor and the collision radius - due to the inverse relationship between  $f_0$  and the collision radius - the combined effect depends on the relative significance of these two factors. This in turn depends on the rate of change of the energy spectrum function in the vicinity of  $f_0$ . For very small particles the entrainment factor approaches unity and is almost unaffected by changes in particle size. Hence, for such small particles the effective diffusivity increases with increasing particle size.

The effect of molecular diffusivity is not taken into consideration because the particles of concern are too large to be influenced by this kind of molecular motion. Also, as pointed out by Saffman [31], even for fluid particles the effect of the molecular diffusion in turbulent diffusion fields is usually small.

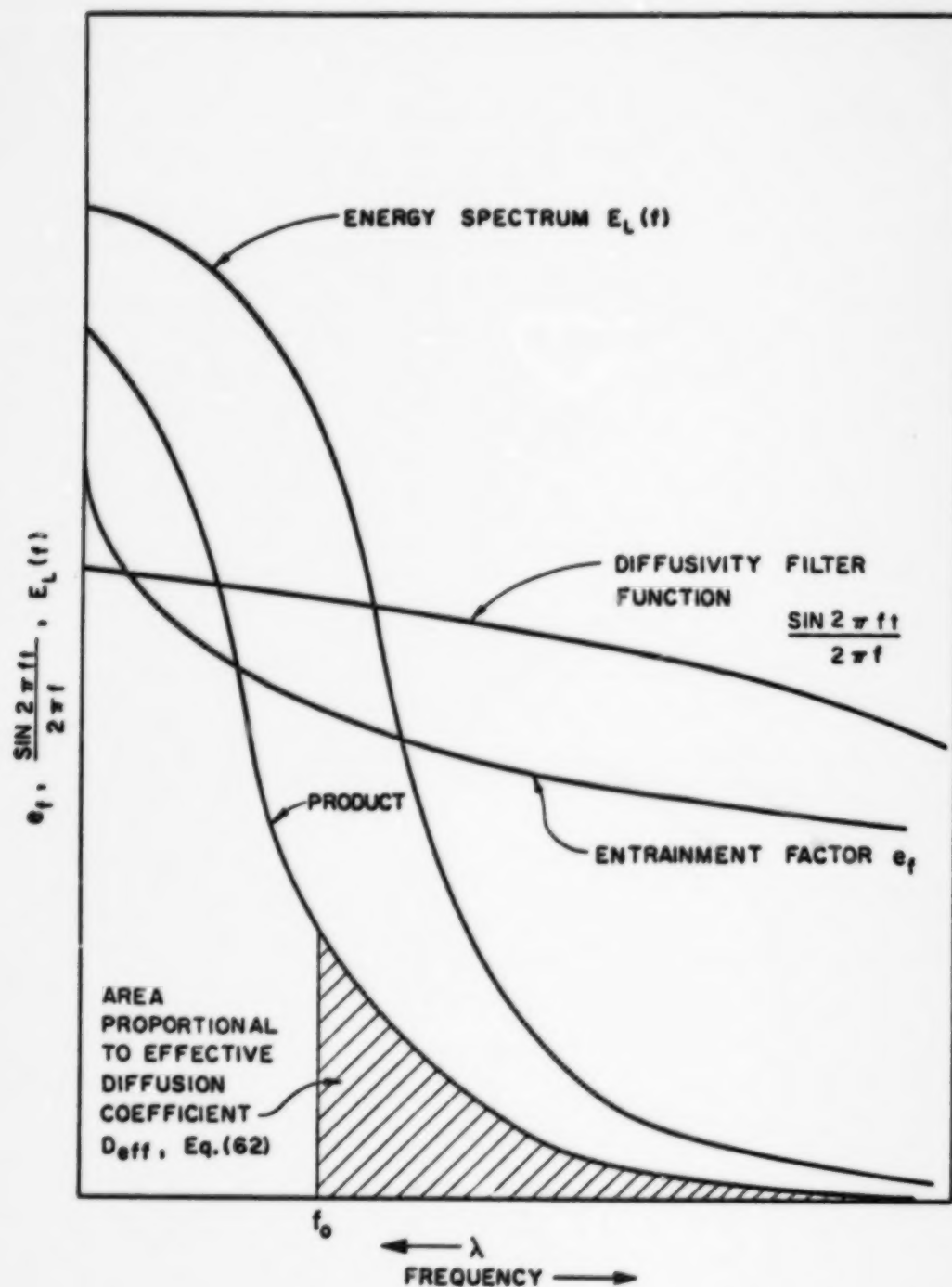


FIGURE 5. ENERGY SPECTRUM AND EFFECTIVE DIFFUSIVITY



### III. THE FLOCCULATION EQUATION

#### FLOC BREAKUP

The flocculation equation in its differential form is an expression for the rate of change in the concentration of particles of a given size. Two opposing processes are responsible for the change in particle concentration, one being the aggregation of particles to form larger flocs and the other the breakup of flocs into smaller fragments. In order to write the flocculation equation a breakup mechanism must be postulated. Unfortunately, floc breakup was not the subject of extensive studies and no breakup theory has received general acceptance. However, plant and laboratory experience provide ample evidence that floc breakup does occur. The existence of an upper limit of floc size and the decrease in flocculation efficiency at very high stirring speeds are two common observations indicating the role of floc breakup. In laminar shear flow, as noted by Thomas [32], flocs will assume rotational motion resulting in a rearrangement of particles within the floc with no significant breakup. However, in turbulent flow, due to rapid changes in velocity and the inertia of the floc particles, disruption will occur. The exact breakup mechanism and the size of the breakup fragments are not known. It is believed, however, that a principal breakup mode producing primary particles is surface erosion of flocs. If such breakup model is assumed, a simple expression for the rate of breakup can be derived. The general expression for the shearing stress in a viscous fluid is given by

$$\tau = \rho C'_d U^2 \quad (64)$$

where  $C'_d$  is a drag coefficient and  $U$  the fluid velocity. In turbulent flow the velocity responsible for floc breakup is of the magnitude of the root mean square velocity fluctuation,  $\sqrt{u^2}$ . Thus, Equation 64 is rewritten as

$$\tau = \rho C'_d \overline{u^2} \quad (65)$$

The rate at which primary particles are stripped from a floc should be proportional to the shearing stress and the surface area of the floc. Thus the rate at which the concentration of primary particles increases due to the breakup of a  $k$ -fold particle is

$$\left( \frac{dn_1}{dt} \right)_k = \frac{4\pi C'_d B'}{R_1^2 v} n_k R_k^2 \overline{u^2} \quad (66)$$

where  $n_1$  is the concentration of primary particles, and  $B'$  is a breakup coefficient. The kinematic viscosity is introduced for dimensional reasons. Defining a generalized breakup coefficient by

$$B = \frac{4\pi C_d B'}{R_1^2 \nu} \quad (67)$$

Equation 66 may be reduced to

$$\left( \frac{dn_k}{dt} \right)_k = B R_k^2 \bar{u}^2 n_k \quad (68)$$

It should be noted that by restricting the foregoing analysis to breakup producing primary particles only it is not implied that all other breakup modes are neglected. The analysis was limited to primary particles merely because the flocculation equation is derived from primary particles. The effect of all breakup modes is considered in subsequent derivations.

#### THE FLOCCULATION EQUATION

The differential form of the flocculation equation can be written as

$$\begin{aligned} \frac{dn_k}{dt} = & \frac{1}{2} \sum_{\substack{i=1 \\ i+j=k}}^{k-1} 4\pi\alpha R_{ij} D_{ij} n_i n_j - \sum_{i=1}^{\infty} 4\pi\alpha R_{ik} D_{ik} n_i n_k \\ & - n_k B_k \bar{u}^2 + \bar{u}^2 \sum_{i=k+1}^{\infty} n_i B_i \quad (69) \end{aligned}$$

The first term in this equation represents the formation of  $k$ -fold particles by collision and aggregation of two smaller particles. The second term is the elimination of  $k$ -fold particles due to their collision with other particles to form larger flocs. The constant  $\alpha$  in these terms is the fraction of collisions resulting in aggregation. The third term is the elimination of  $k$ -fold particles by their breakup, and the last term represents formation of  $k$ -fold particles by breakup of larger particles.  $B_k$  and  $B_i$  are breakup functions which for the general case are unknown. As the performance of a flocculation reactor is most simply measured by the reduction of primary particles ( $k = 1$ ), the flocculation equation for primary particles is of most significance. Fortunately the equation becomes considerably simpler than Equation 69 as the first and third terms vanish.

Using the surface erosion breakup model of the previous section, the flocculation equation for primary particles becomes

$$\frac{dn_1}{dt} = -4\pi\alpha \sum_{i=1}^{\infty} R_{1i} D_{eff} n_i n_1 + B \overline{u^2} \sum_{i=2}^{\infty} n_i R_i^2 \quad (70)$$

Following the procedure used by Swift and Friedlander [17] or Wang [18], a solution to this differential equation may be found by replacing the summations by integrals and seeking the solution for the resulting integro-differential equation. This tedious and somewhat questionable procedure can be avoided by assuming a simplified particle size distribution. Based on the experimental observations of the current study, a continuous flow flocculation system is characterized by a bimodal particle size distribution consisting largely of primary particles (or flocs with relatively few particles) and flocs comprised of very large numbers of primary particles. Variation in size within each of these groups is small compared to the difference in size between the groups. For such a simplified floc size distribution the flocculation equation becomes

$$\frac{dn_1}{dt} = -4\pi\alpha R_{1F} D_{eff} n_1 n_F + B n_F R_F^2 \overline{u^2} \quad (71)$$

where the subscripts 1 and F refer to primary particles and flocs, respectively.

#### THE FLOCCULATION EQUATION FOR CONTINUOUS FLOW SYSTEMS

In principle, Equation 71 can be solved by integration of both sides. However, this is not an easy task as  $R_{1F}$ ,  $D_{eff}$ ,  $n_F$ ,  $n_1$ , and  $R_F$  are all time dependent. It also turns out that this solution is not of great interest as it applies to batch processes, whereas in practice flocculation is almost exclusively performed in continuous flow systems. Applying Equation 71 to a completely mixed single compartment continuous flow tank reactor (CSTR) at steady state, one obtains the following mass balance equation:

$$n_1^0 - n_1^1 = \left( 4\pi\alpha R_{1F} D_{eff} n_1^1 n_F - B \overline{u^2} n_F R_F^2 \right) T \quad (72)$$

where  $n_1^0$  is the concentration of primary particles entering the system,  $n_1^1$  is the concentration in the effluent from the first (and in this case the only) reactor, and  $T$  is the mean residence time of the system. The collision radius  $R_{1F}$  may be estimated by  $R_1 + R_F$ , and as the floc is usually much larger than the primary particle, one may assume that  $R_{1F} \approx R_F$ . With this assumption Equation 72 can be rearranged to read

$$\frac{n_1^0}{n_1^1} = \frac{1 + 4\pi\alpha R_F D_{eff} n_F T}{1 + Bu^2 R_F^2 \frac{n_F}{n_1^0} T} \quad (73)$$

$n_1^0/n_1^1$  will be referred to as the "performance parameter." Equation 73 describes the relation between all the physical parameters involved in the flocculation process. Two of these parameters,  $T$  and  $u^2$ , are determined by design decisions. The others are dependent on the conditions of operation and are less clearly controllable. In order to arrive at a more convenient equation for practical use, several assumptions must be made.

Mass balance on the suspended solids requires that

$$n_F = \frac{\rho_1 R_1^3 (n_1^0 - n_1^1)}{\rho_F R_F^3} \quad (74)$$

where  $\rho_1$  and  $\rho_F$  are the densities of primary particles and flocs, respectively. For good performance, i.e.,  $n_1^0 \gg n_1^1$  and with the assumption that high performance ( $n_1^0 \gg n_1^1$ ) is accompanied by larger floc density, the number concentration of flocs can be approximated by

$$n_F = K_1 n_1^0 \frac{R_1^3}{R_F^3} \quad (75)$$

where  $K_1 = \rho_1/\rho_F (n_1^0 - n_1^1)/n_1^0$ . The floc size is controlled by floc breakup which, in turn, is a function of the mean square velocity fluctuation. Thus, one relationship might be

$$R_F = K_2 \left( \overline{u^2} \right)^{-K_3} \quad (76)$$

Experimental results described in Chapter IV indicate that  $K_3$  is close to unity and that a nearly linear relationship does exist.

The effective diffusion coefficient is a function of both the energy spectrum and the floc size. The relationship between these three parameters is given by Equation 62. This equation cannot be

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\*It is recognized that the number and volume of flocs are closely related to the coagulant dose. This is not reflected in Equations 74 and 75 or in the assumptions leading to their derivation. However, these equations and assumptions are not necessary for the subsequent derivations. An alternative approach leading to the same final results is available.

reduced to a simpler and reliable form. However, based on Figure 5 some approximate properties of the effective diffusion coefficient may be derived. First, it should be noticed that the lower limits of the two integrals in Equation 62 are the same. This limit is proportional to  $f_0$  which in turn is inversely proportional to the floc size,  $R_F$ . The two integrals differ only by the entrainment factor. If  $R_1$  and  $R_2$  in Equation 62 are taken as  $R_1$  and  $R_F$ , the entrainment factor for the primary particle will be much larger than that for the floc since  $R_F \gg R_1$ . Thus, as a first approximation, the effective diffusivity may be estimated by

$$D_{eff} = \int_{f_0}^{\infty} E_L(f) \frac{\sin 2\pi f t}{2\pi f} \cdot \frac{A f^{-\frac{2}{3}}}{2R_1 + A f^{-\frac{2}{3}}} df \quad (77)$$

This integral can be evaluated by measuring the shaded area of Figure 5. A simpler expression for  $D_{eff}$  can be obtained as follows: For short diffusion times the diffusivity filter function may be replaced by a characteristic diffusion time,  $\tau_0$ . An average entrainment factor,  $\bar{e}_f$  may replace the third term in the integral of Equation 77 which then can be rewritten as

$$D_{eff} = \tau_0 \bar{e}_f \int_{f_0}^{\infty} E_L(f) df \quad (78)$$

Thus, the effective diffusivity is a function of a typical diffusion time  $\tau_0$ , an average entrainment coefficient  $\bar{e}_f$ , and the integral of the spectrum function from  $f_0$  to  $\infty$ . Noting that  $\int_0^{\infty} E_L(f) df = \overline{u^2}$ , the integral of Equation 78 can be replaced by  $K_4 \overline{u^2}$  where  $K_4$  is the fraction of  $\overline{u^2}$  contributed by fluid motions of frequencies higher than  $f_0$ . Finally, one can argue that  $K_4$  is proportional to some function of  $f_0$  which in turn depends on  $R_F$ . If the area under the spectrum curve for  $f > f_0$  is approximated by a triangle the horizontal side of which is proportional to  $R_F$ , then  $K_4$  can be replaced by  $K_5 R_F^2$  where  $K_5$  is a dimensional constant which reflects the properties of the spectrum function in the region  $f > f_0$ . With these assumptions the effective diffusivity can be approximated by

$$D_{eff} = K_S \overline{u^2} R_F^2 \quad (78a)$$

where  $K_S = K_5 \bar{e}_f \tau_0$ . The constant  $K_S$  depends primarily on the shape of the energy spectrum function and is therefore referred to as the "spectrum factor."

With Equations 73 through 78, the following expression for the performance parameter is obtained:



$$\frac{n_1^0}{n_1^1} = \frac{1 + 4\pi\alpha K_1 K_s R_1^3 n_1^0 T \overline{u^2}}{1 + B \frac{K_1}{K_2} R_1^3 T (\overline{u^2})^{K_3+1}} \quad (79)$$

#### MEAN SQUARE VELOCITY FLUCTUATIONS AND POWER INPUT

Although  $\overline{u^2}$  is a well defined property of the turbulence, its experimental determination is somewhat more involved. On the other hand, the total power dissipated in the fluid is easily determined as it simply equals the net power input. For this reason the total power input is very often used in water purification practice as the parameter representing the turbulence. To express the flocculation performance parameter in terms of the power input, a relationship between the mean square velocity fluctuation and the power dissipation is required. This subject was treated by Kolmogoroff, Heisenberg and others and is discussed extensively by Batchelor [33]. When turbulence is generated in a fluid by an external source, the directly produced eddies are usually of large scale, comparable to the dimensions of the turbulence generating device. Inertial interaction between these eddies results in energy transfer to smaller and smaller eddies. Not all the energy is transferred, however, as part is dissipated by viscous forces and is converted into heat. The rate of dissipation depends on the ratio of the inertia to viscous forces, i.e., on the Reynolds number. Each fraction of the spectrum has a characteristic scale and velocity, with a characteristic Reynolds number given by

$$R_e = \frac{v \lambda}{\nu} \quad (80)$$

where  $\lambda$  is the eddy scale;  $v$ , its velocity, and  $\nu$  the kinematic viscosity of the fluid.

For very high Reynolds numbers, i.e., large eddies or small frequencies, the dissipation is negligible. The dissipation becomes significant when the Reynolds number is of the order of unity. According to Kolmogoroff, there is a range of high frequencies where the turbulence is statistically in equilibrium and uniquely determined by the energy dissipation rate,  $\epsilon$ , and the kinematic viscosity,  $\nu$ . From dimensional reasoning the length and velocity scale in this region are given by

$$\lambda_0 = \left( \frac{\nu^3}{\epsilon} \right)^{\frac{1}{4}} \quad (81)$$

and

$$v_0 = (\nu \epsilon)^{\frac{1}{4}} \quad (82)$$

These length and velocity scales correspond to a Reynolds number of unity. As in this region the turbulence depends only on  $\nu$  and  $\epsilon$  it was possible to derive a theoretical expression for the spectrum. Thus, in the universal equilibrium range the energy spectrum is given by

$$E(f) = K \epsilon^{\frac{2}{3}} f^{-\frac{5}{3}} . \quad (83)$$

For frequencies below the equilibrium region, the energy spectrum depends on the turbulence generating mechanism. From Equation 83 one can calculate the mean square velocity fluctuations produced by frequencies which are within the equilibrium region. Since there is no reason to believe that flocculation is unaffected by motions of frequencies outside this region, Equation 83 does not provide the answer to our basic problem. The energy spectrum outside the equilibrium region depends on the mechanism by which the turbulence energy is transferred to the fluid. This leads to the obvious conclusion that with equal power input, different stirring devices may produce different energy spectra. In principle a theoretical relationship between dissipation and the entire energy spectrum can be derived if the energy transfer function of the turbulence generating device is known. This information, however, is generally not available.

To obtain the necessary relationship between the mean square velocity fluctuations and the power dissipation the assumption is made that

$$\overline{u^2} = K_p G . \quad (84)$$

This assumption is supported by experimental observations described in Chapter V. It should be noted that in the equilibrium range,  $u^2$  is proportional to  $\epsilon^{\frac{2}{3}}$ , or to  $G^{\frac{4}{3}}$ , by virtue of Equation 4. Outside the equilibrium range the exact dependence of  $u^2$  on  $\epsilon$  is unknown, and a linear dependence was assumed for the entire range. The proportionality constant,  $K_p$ , reflects the ability of a turbulence generating device of producing high velocity fluctuations for a given power input. It is therefore referred to as the "stirrer performance coefficient."

#### THE WORKING EQUATION

Making use of Equations 79 and 84, and assuming  $K_3 = 1$ , the following working equation for the performance parameter of a single flocculation reactor is obtained

$$\frac{n_1^0}{n_1} = \frac{1 + K_F K_S K_P n_1^0 T G}{1 + K_B T G^2} \quad (85)$$

where the flocculation and breakup coefficients are defined by

$$K_F = 4\pi\alpha K_1 R_1^3 \quad (86)$$

and

$$K_B = B K_P^2 R_1^3 \frac{K_1}{K_2} \quad (87)$$

and  $K_S$  and  $K_P$  are the spectrum and stirrer performance coefficients, respectively. Equation 85 can be applied to a multicompartment system. If the overall residence time is  $T$ , and the number of equal-volume compartments connected in series is  $m$ , the performance parameter of the  $i$ -th compartment is given by

$$\frac{n_1^{i-1}}{n_1^i} = \frac{1 + K_F^i K_P^i K_S^i n_1^0 G^i \frac{T}{m}}{1 + K_B^i \frac{n_1^0}{n_1^{i-1}} (G^i)^2 \frac{T}{m}} \quad (88)$$

The superscript,  $i$ , indicating the  $i$ -th compartment, is added to each of the constants as well as  $G$  for generality, as these are not necessarily the same for each of the compartments in a series.

#### SUMMARY

In this chapter a theoretical rate equation for the flocculation process was developed. The equation was derived for a completely mixed continuous flow reactor in which aggregation and breakup occur simultaneously. The aggregation term was obtained from the collision frequency assuming a fixed aggregation-collision ratio. The breakup term was based on the assumption that primary particles are formed by surface erosion of flocs. The breakup of flocs into fragments of various other sizes was accounted for by assuming an inverse relationship between floc size and power input. The resulting equation (Equation 73) provided the relationship between the performance of a single flocculation reactor and such parameters as residence time distribution, number concentration of flocs and primary particles, floc size, effective diffusion coefficient, and mean square velocity fluctuations. The equation was then reduced to a working form by making the following assumptions:

1. The effective diffusivity is directly proportional to the spectrum coefficient, the mean square velocity fluctuation, the square of the floc radius, and an entrainment factor which is inversely proportional to the primary particle size.

- ii. The number concentration of flocs is directly proportional to the total volume of primary particles in the coagulated water, and inversely proportional to the volume of a single floc.
- iii. The floc radius is inversely proportional to the mean square velocity fluctuations.
- iv. The mean square velocity fluctuations is directly proportional to the root-mean-square velocity gradient, i.e.,  $G$ .

These assumptions were based on theoretical arguments and substantiated by experimental observations. It is recognized, however, that the validity of at least some of the assumptions is questionable, but were necessary to arrive at a working equation.

To summarize, the flocculation equation in its working form reads

$$\frac{n_1^0}{n_1^1} = \frac{1 + K_F K_S K_P n_1^0 T G}{1 + K_B T G^2} \quad (85)$$

where  $K_F$  and  $K_B$  are the flocculation and breakup coefficients,  $K_S$  and  $K_P$  are the spectrum and stirrer performance coefficients,  $T$  is the residence time, and  $G$  is the root-mean-square velocity gradient.

#### IV. EXPERIMENTAL INVESTIGATION OF FLOCCULATION KINETICS

##### OBJECTIVES OF EXPERIMENTAL WORK

In previous chapters a working equation describing the flocculation process was derived. It was based on a theoretical expression for the rate of collisions between suspended particles in a turbulent fluid. It has been shown that the efficiency of the flocculation process depends on the kinetic energy of the turbulence, its spectrum, the residence time, and the concentration of flocs and primary particles. The general objective of the experimental work was to test the validity of the proposed theory and to evaluate the numerical constants which the theory fails to provide. It should be appreciated that the theoretical and experimental phases of this study were carried out simultaneously. Some of the assumptions made in the previous theoretical analysis were in fact based on experimental observations which will be discussed in the following sections. Hence, the arguments presented previously are partially empirical. It should also be noted that due to limitations in the number and accuracy of the experiments, some of the parameters could be analyzed qualitatively only.

The experimental work was composed of two main phases. The first was concerned with the efficiency of a laboratory scale flocculation reactor in carrying out its assigned task and the other was devoted to an investigation of the turbulence field in the reactor. The experimental procedures, the apparatus, and the results are described in this chapter and are tied to the theoretical work presented earlier.

##### APPARATUS

The experimental work was conducted using the reactor shown in Figure 6. The system was designed to operate as a series of four continuous stirred-tank reactors. The water supply system and variable speed stirring paddles were designed to permit operation over a wide range of residence times and power inputs. The reactor was constructed of acrylic plastic and supported by an aluminum frame. The inside dimensions of each of the four compartments were 10 x 10 inches. Water depth in the reactors was controlled by a 90° weir of adjustable elevation and was kept at 10 inches throughout the entire experimental work. The flow rate was controlled by a constant head overflow weir and measured by a Fischer and Porter Flowrator. The turbid raw waters were stored in a 1000 gallon polyethylene lined redwood tank to assure a constant raw water quality over a number of runs. The raw water and coagulant were introduced through a flash mixer having a fixed residence time of 30 seconds. Stainless steel stirrers were mounted on a vertical shaft and were rotated in a horizontal plane by a 1/8 hp DC motor, a product of the Bodine Electric Company, and a Minaric speed control. Provisions were made for connecting a torque meter to one of the stirrer



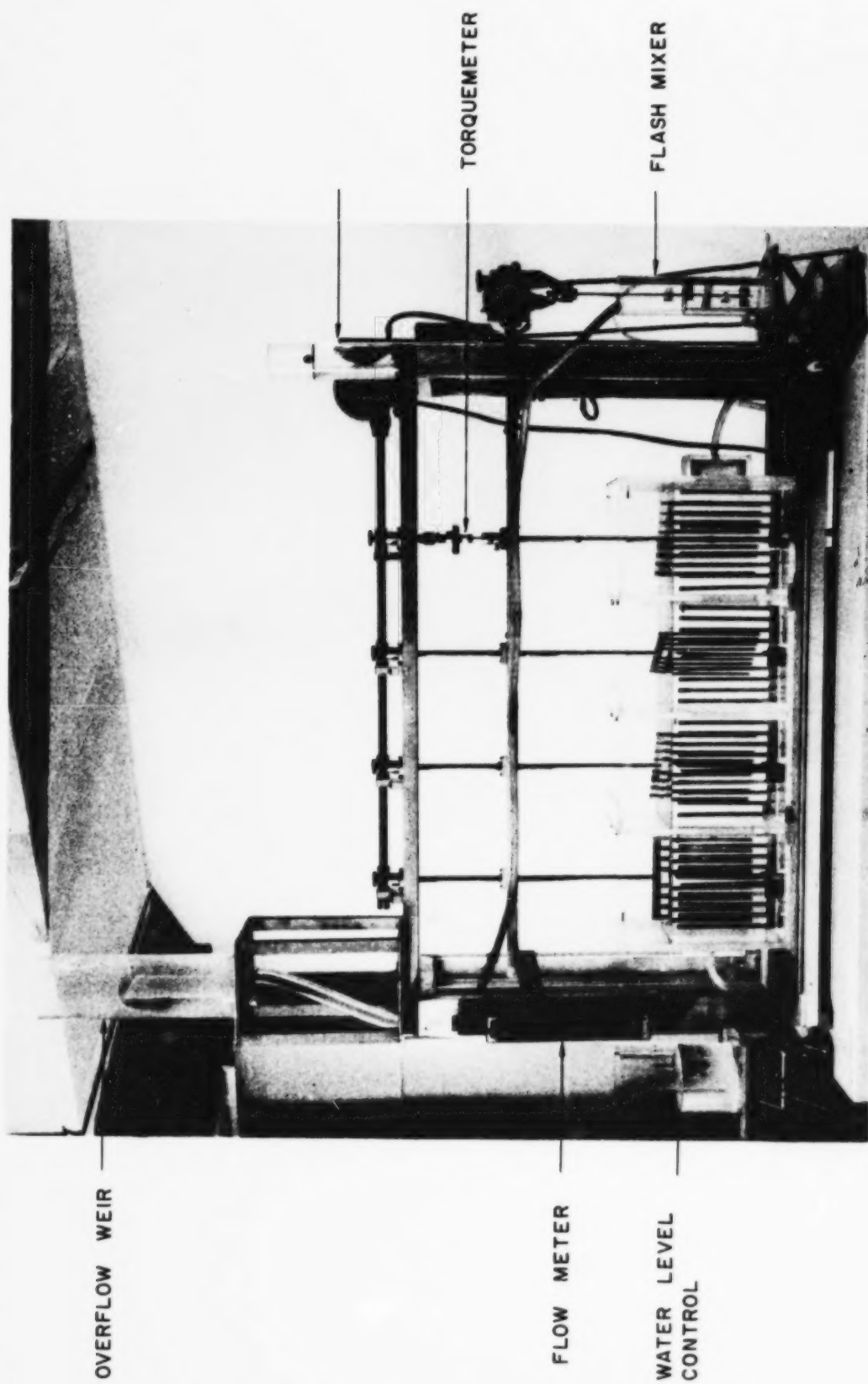


FIGURE 6. LABORATORY - SCALE FLOCCULATOR

shafts so that the power input could be determined. Two different stirring devices were used; one of the turbine type and one of the "stake and stator" type. These are shown in Figure 7.

Five ports for introducing a hot-film anemometer probe for turbulence measurements were provided at the bottom of one compartment. The electronics associated with turbulence measurements consisted of a constant temperature heat flux system, a hot-film anemometer probe, a root mean square voltmeter and an analog-digital converter recorder. These are discussed in more detail in Chapter V.

#### EXPERIMENTAL PROCEDURES - FLOCCULATION TESTS

Raw waters for flocculation tests were prepared from East Bay Municipal Utility District water supply system to which 25 mg/l of kaolin powder were added. The kaolin, a product of Mallinckrodt Chemical Works, was suspended in several liters of distilled water for a period of at least 24 hours so that aggregates were broken and a fairly homogeneous suspension of clay particles was produced. The particle size distribution analysis of the kaolin powder was carried out by the standard settling test employing Stokes law with the results shown in Figure 8. The raw suspension of 25 mg/l kaolin in tap water was stored in a 1000 gallon tank under continuous gentle mixing. From the storage tank the water was pumped into a constant head tank from which it passed by gravity through a flowmeter into the flash mixer. The coagulant selected for this investigation was common filter alum. A stock solution of 1000 mg/l was prepared by dissolving granular aluminum sulfate,  $\text{Al}_2(\text{SO}_4)_3 \cdot 14 \text{H}_2\text{O}$  of technical grade in distilled water. The alum solution was pumped into a constant head tank and passed by gravity through a flowmeter into the flash mixer. Based on preliminary jar tests alum flow rate was adjusted to give a constant dosage of 25 mg/l. The two streams, turbid water and alum solution, were mixed for 30 seconds in the flash mixer, from which they passed into the flocculation reactor. Flow rates and the water elevation in the reactor were adjusted to give the predetermined residence time. The flowmeters were calibrated volumetrically prior to these experiments. The rotating speed of the stirring paddles was adjusted to produce the desired power input.

The degree of completion of the flocculation process was measured following Harris' [34] procedure by comparing the steady state concentrations of primary particles before and after flocculation. Samples for analysis were withdrawn from the reactor by carefully dipping a 100 ml flask into the reactor. A set of three duplicate samples were taken for each analysis. For the determination of primary particle concentration in the raw water the samples were vigorously shaken so that all flocs were broken to form a homogeneous suspension of primary particles. Three additional samples were withdrawn at the same time and were allowed to settle in a test tube for 30 minutes. The settling tubes were submerged in a water bath through which the effluent from the reactor was flowing so as to avoid thermal density currents during settling. After settling, the supernatant was withdrawn through side arms in the settling tubes for analysis. The concentration of primary

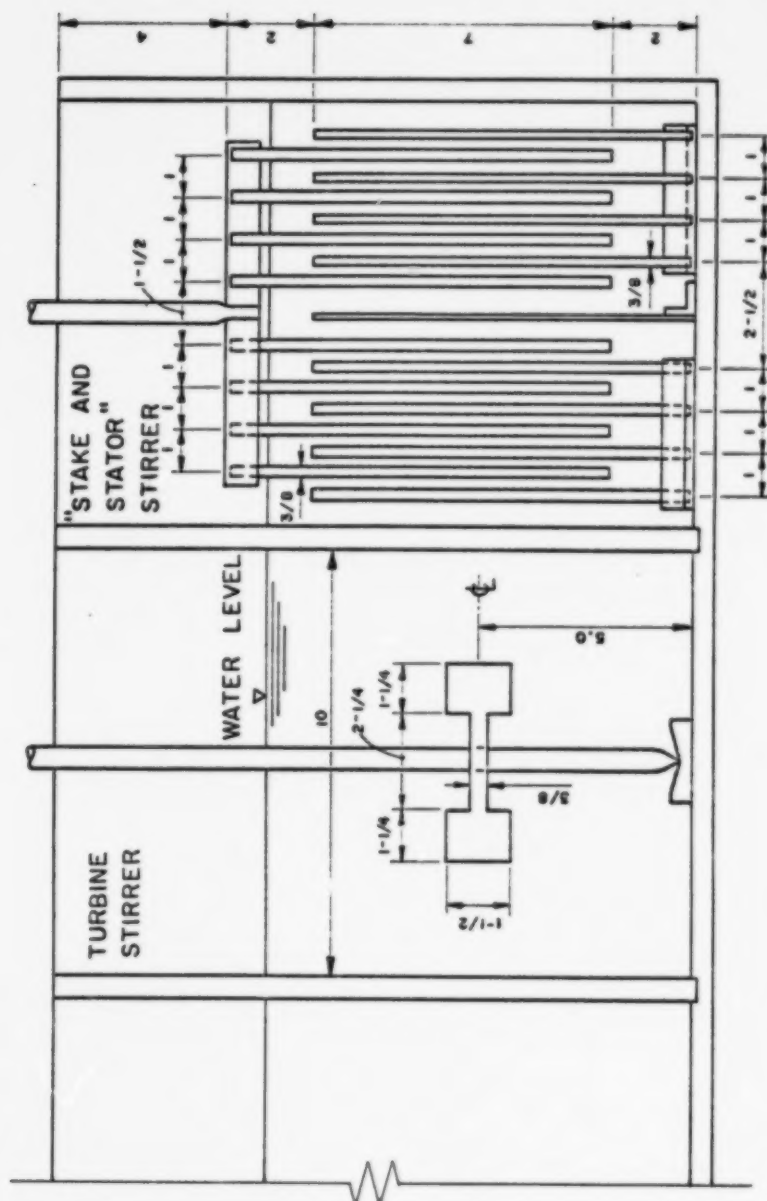


FIGURE 7. STIRRERS

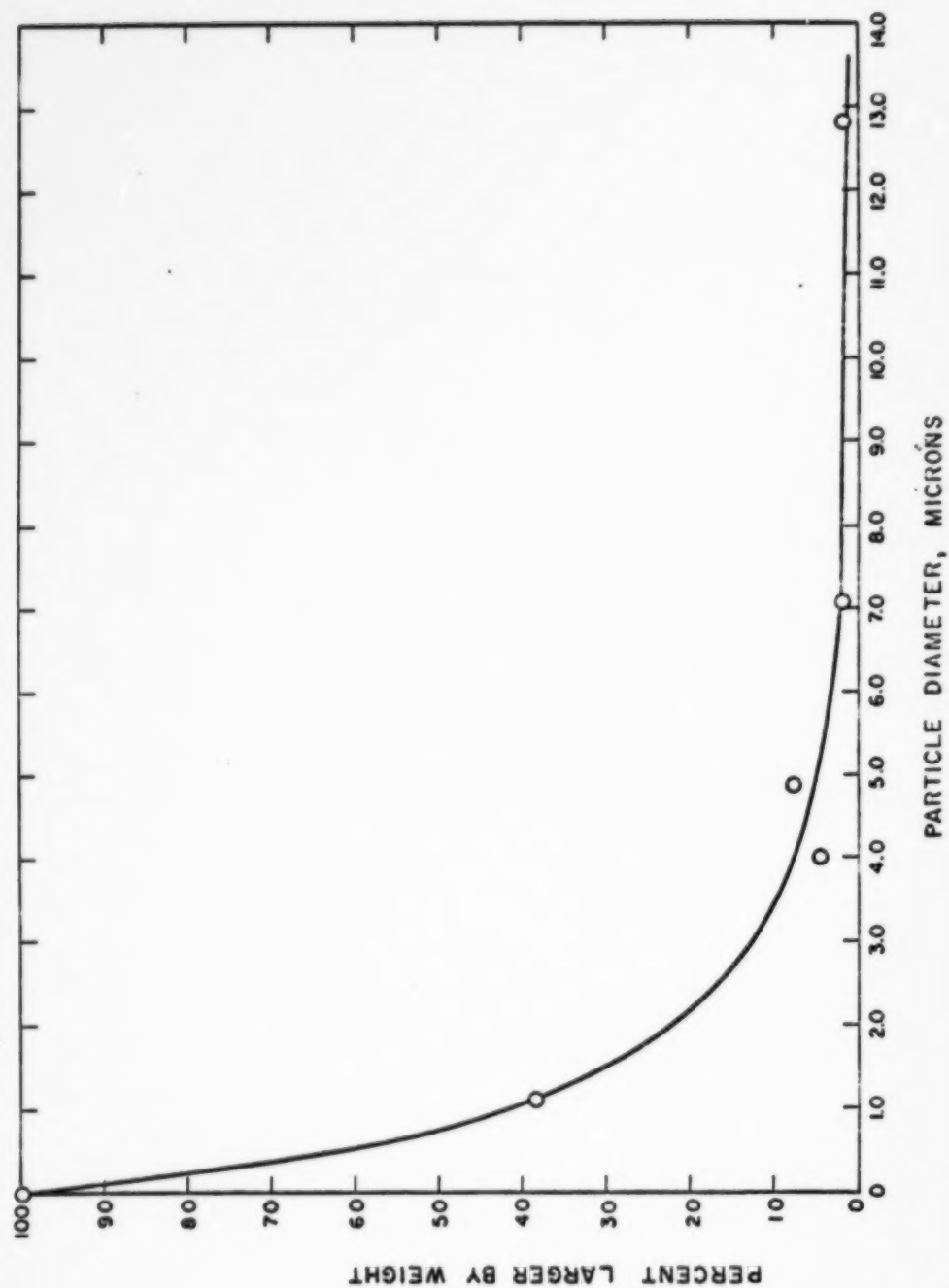


FIGURE 8. PARTICLE SIZE DISTRIBUTION OF KAOLIN CLAY

particles in the supernatant represented the unflocculated fraction of the coagulated suspension. Previous tests have shown that within the 30 minutes allowed for settling there was practically no settling of primary particles in the test tube, whereas most flocs settled. This settling procedure was observed throughout the entire investigation.

The relative concentration of primary particles in the samples was determined by means of the Model 2000 Universal Light Scattering Photometer manufactured by the Phoenix Precision Instrument Company. The instrument enables determination of the absolute turbidity of a sample. A collimated light beam generated by a mercury lamp was directed toward one side of a rectangular test cell containing the sample. The intensities of the transmitted and 90° scattered lights were measured by a photomultiplier tube. The absolute turbidity is a linear function of the ratio of scattered to transmitted light. The instrument was calibrated using standard kaolin suspensions of known concentrations and the calibration curve is given in Figure 9. During the first flocculation runs duplicate samples of both flocculated and unflocculated suspensions were analyzed for suspended solids by the light scattering photometer and the technique suggested by Standard Methods [35]. The results were practically the same. Similar tests were conducted on colored samples and no effect of color on the suspended solid determination was shown by the Light Scattering Photometer at color intensities encountered in the research.

The stirring power input to the reactor was determined experimentally by direct measurement of the torque on the paddle shaft at various rotation speeds. The torquemeter, a product of Power Instruments Inc., was first calibrated by the application of known stationary torques. The power transmitted by the shaft is composed of the power transferred to the water and the power lost in the bearings due to friction. Torque measurements were carried out with water at normal operation level and with the reactors empty. The net power transferred to the fluid was calculated by subtracting the latter from the former. The relation between power, torque, and rotation speed in general is given by

$$P = \tau \omega \quad (89)$$

where  $P$  is the power input,  $\tau$  is the torque, and  $\omega$  is the angular velocity of the rotating paddle. The power input per unit mass of fluid was calculated by

$$\epsilon = \frac{P_t - P_o}{\rho V} \quad (90)$$

where  $\epsilon$  is the power input per unit mass of fluid,  $P_t$  and  $P_o$  are the power transmitted to the paddle shaft with water at normal operating level and with empty tank, respectively,  $\rho$  is the water density and  $V$  is the tank volume. The absolute velocity gradient,  $G$ , was calculated by Equation 4. Water density was taken as 1.00 g/cm<sup>3</sup> and kinematic



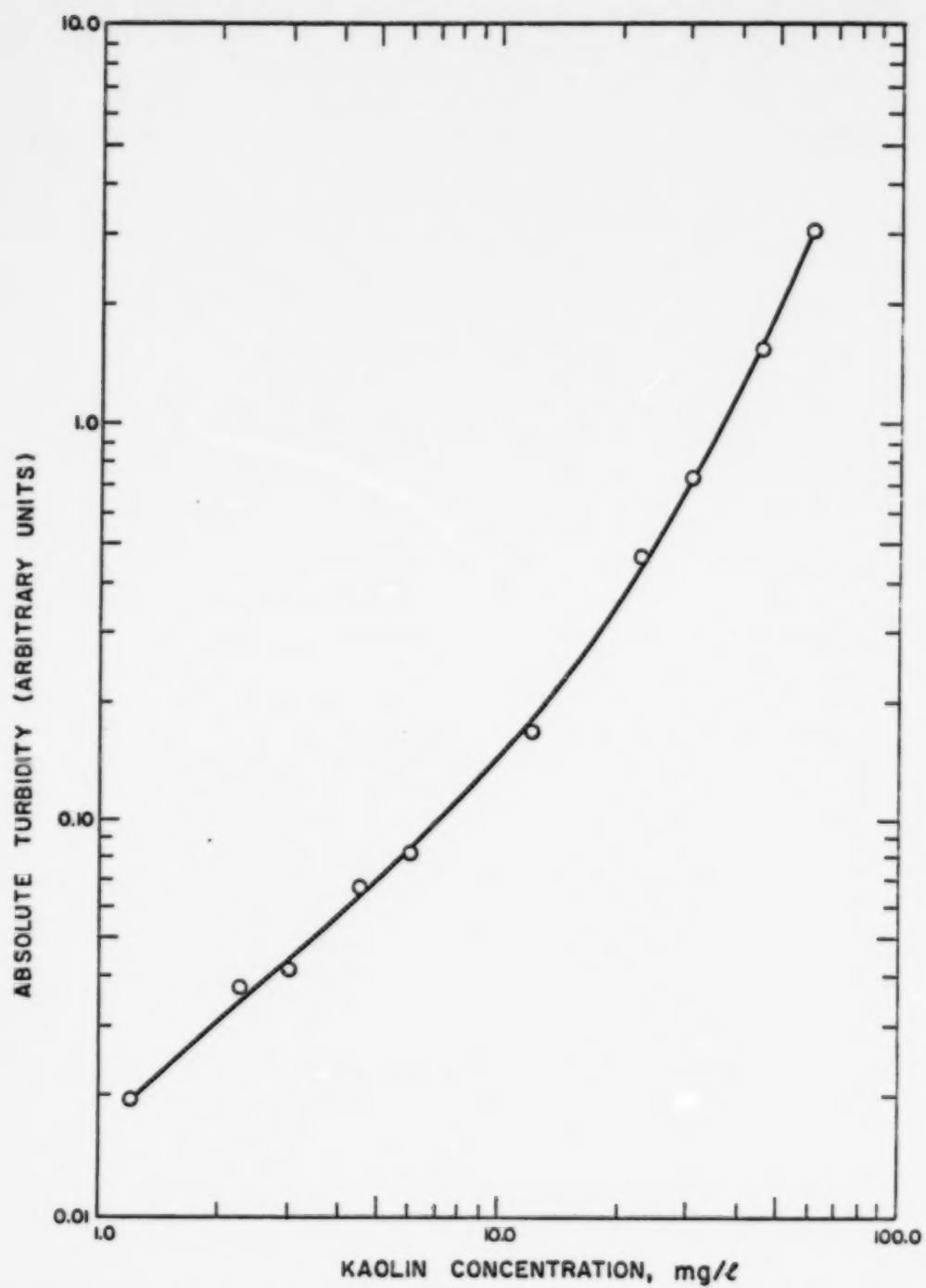


FIGURE 9. CALIBRATION CURVE FOR LIGHT-SCATTERING PHOTOMETER

viscosity as  $1.002 \times 10^{-2}$  Stokes [36]. Water temperature was  $20^\circ \pm 1^\circ\text{C}$  throughout the experimental work. Calibration curves showing the relation between paddle rotation speed and the absolute velocity gradients are given in Figure 10. The experimental results can be compared to those predicted by theory as follows: the force acting on an immersed body moving in a fluid is given by [37]

$$F = C_D \rho \frac{v^2}{2} A \quad (91)$$

where  $F$  is the force acting on the body,  $\rho$  is the fluid density,  $v$  is the velocity of the moving body relative to the surrounding fluid, and  $A$  is the surface area of the moving body normal to the direction of its motion. Assuming a fixed ratio between the velocity of the paddles relative to the water and the paddle's rotation speed, the power on the rotating paddles can be expressed as

$$P = F v = K' N^3 \quad (92)$$

where  $K'$  is a constant and  $N$  is the rotation speed of the paddles. From Equations 4 and 92, the root-mean-square velocity gradient may be related to the rotation speed by

$$G = K N^{\frac{3}{2}} \quad (93)$$

where  $K$  is a constant.

Thus when  $G$  is plotted versus  $N$  on a logarithmic scale, theory predicts a straight-line relationship with a slope of 1.50. The experimental results (Figure 10) show that for both paddle types the points indeed fall on a straight-line, but with slopes of 1.54 for the turbine paddle and 1.61 for the stake and stator paddle.

#### RESULTS OF THE FLOCCULATION TESTS

The first series of flocculation tests were limited to a single compartment reactor. Four different residence times ranging from 8 to 24 minutes were employed. Energy inputs expressed in terms of root-mean-square velocity gradients were varied at each residence time in seven steps from 30 to  $240 \text{ sec}^{-1}$ . The same series of experiments were run with each of the two stirring devices. Most experiments were repeated several times to give more meaningful results from a statistical point of view. The results are shown in Figures 11 through 14, and tabulated in Appendix B. The flocculation performance parameter shown in the figures is  $n_0/n_1$  which appears in the theoretical flocculation equation (Equation 85). The plots contain all actual data points as well as the average values through which the connecting lines were drawn.

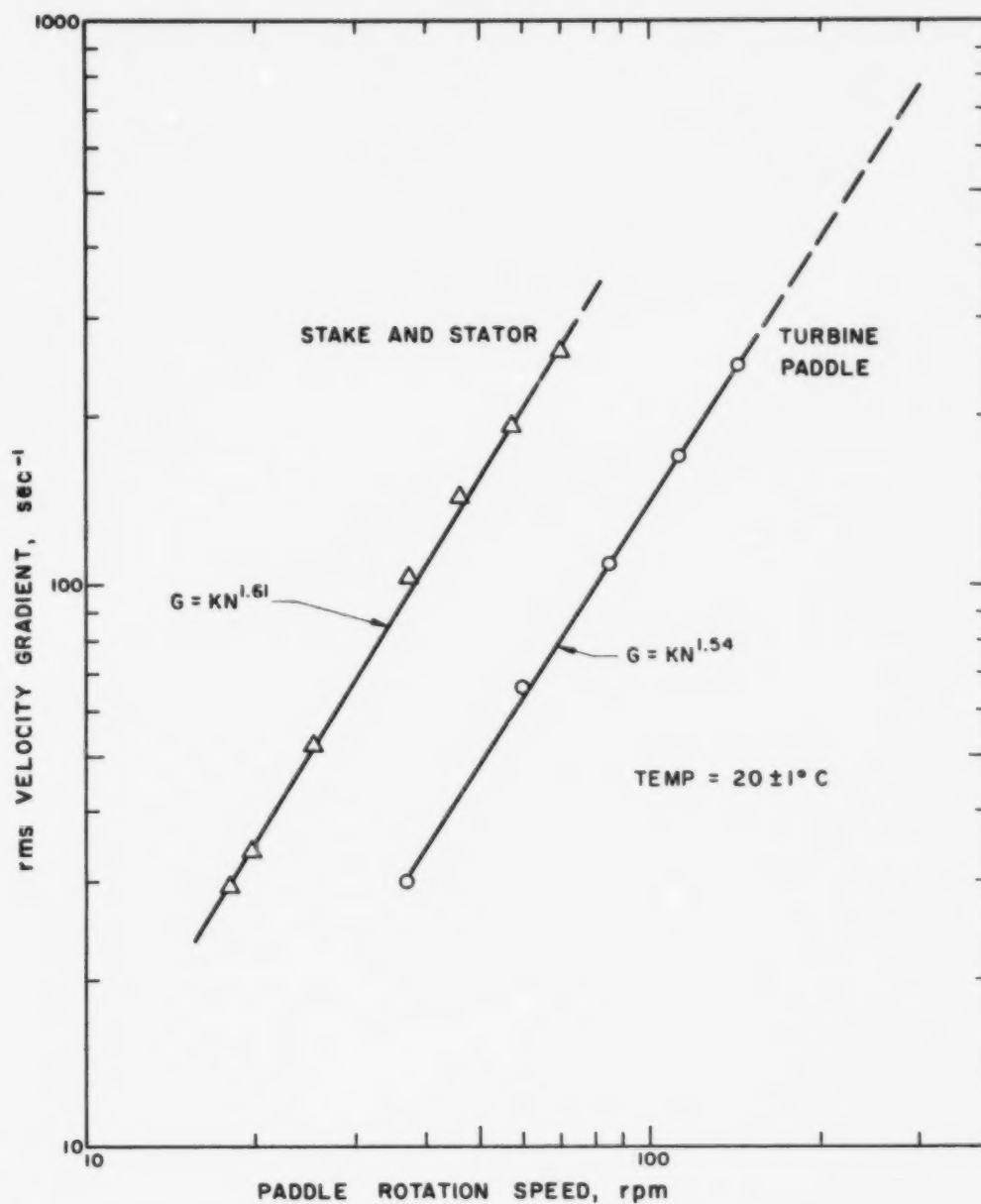
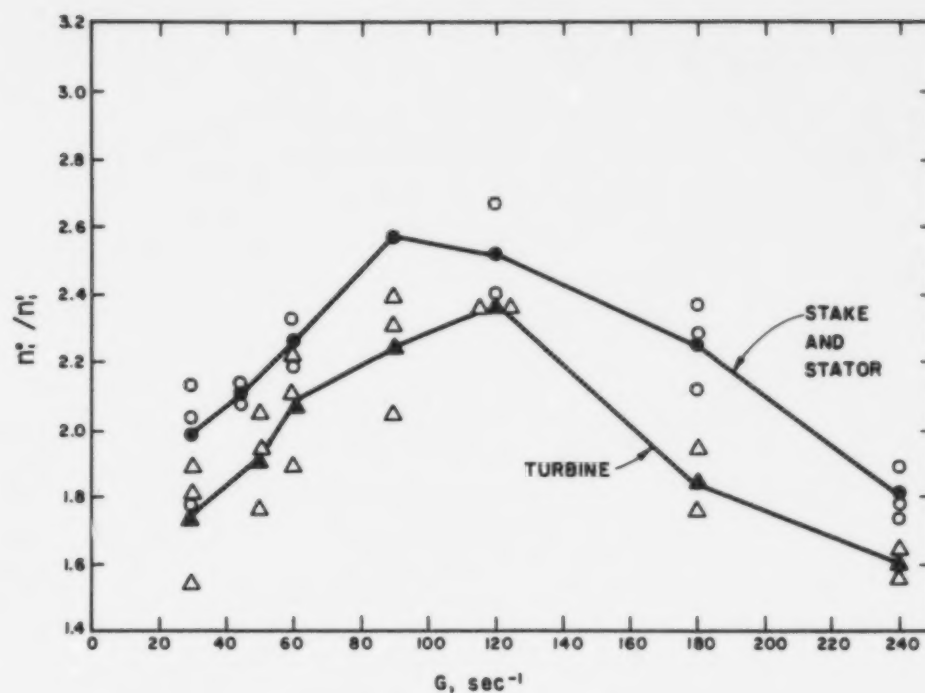
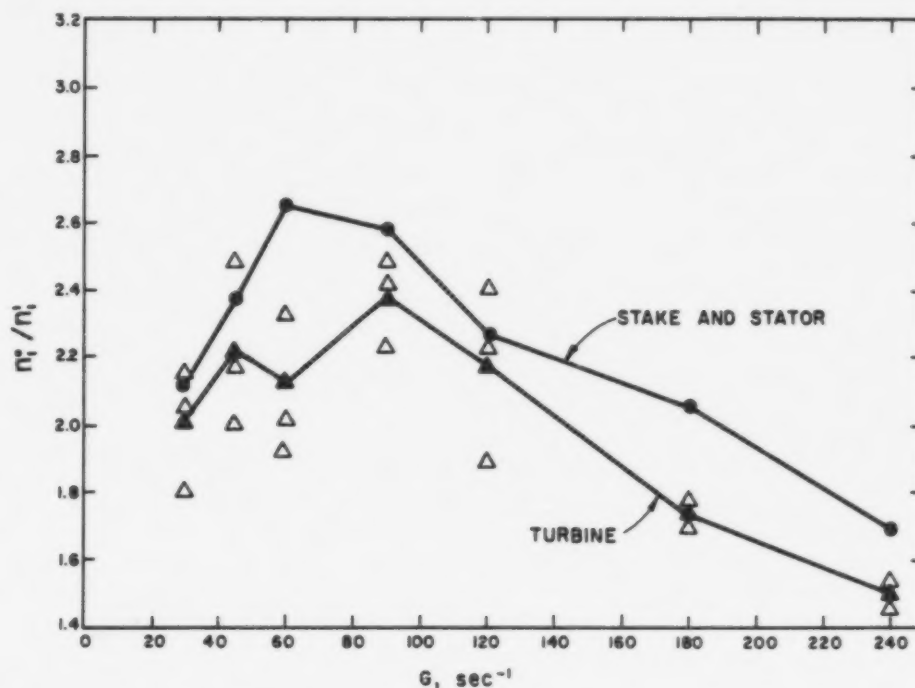
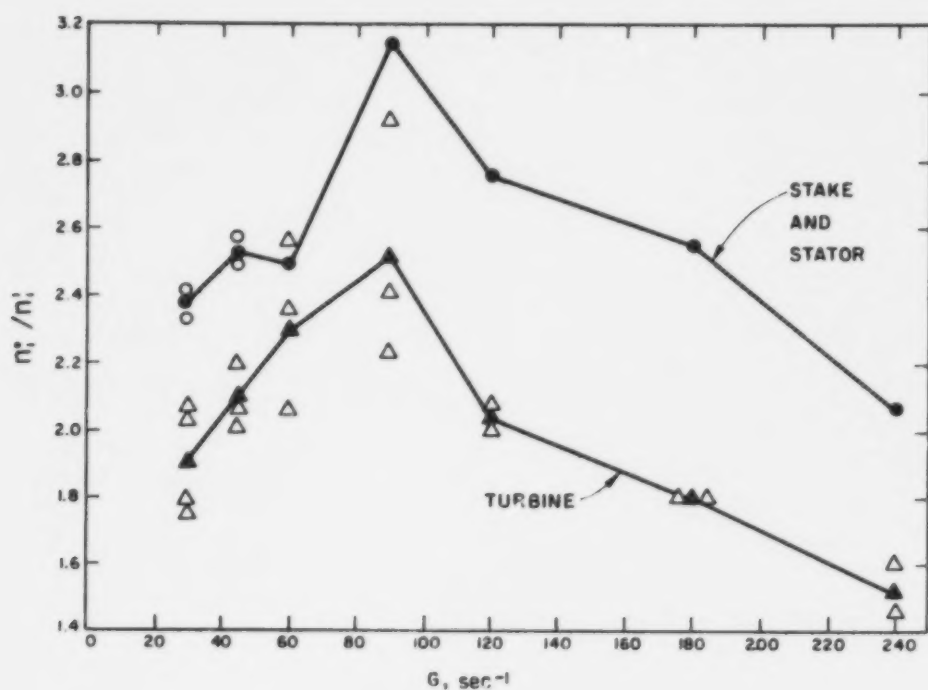
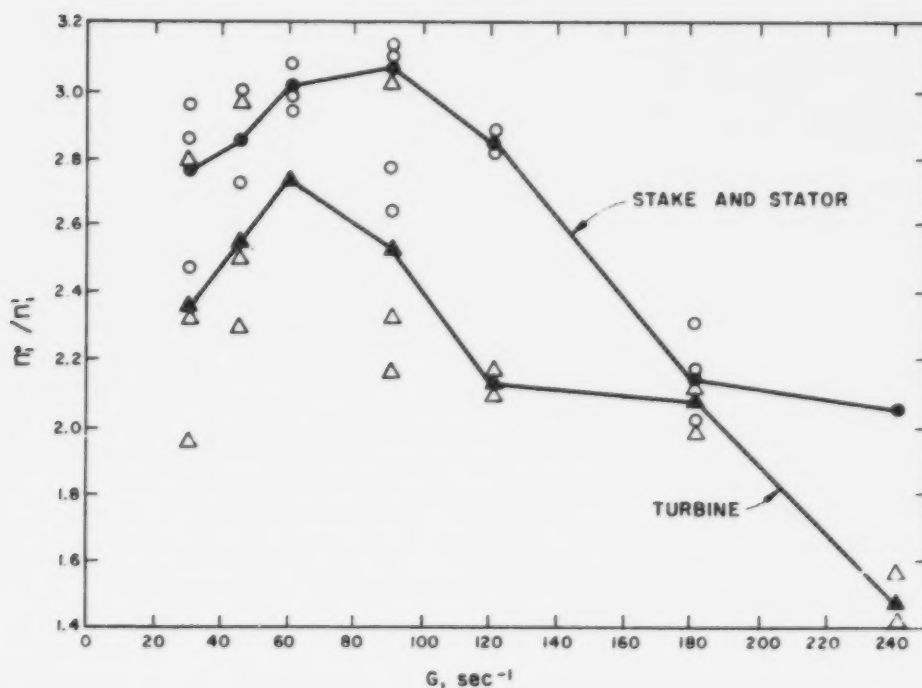


FIGURE 10. VELOCITY GRADIENT VERSUS PADDLE ROTATION SPEED

FIGURE 11. PERFORMANCE OF A SINGLE REACTOR AT  $T = 8$  minFIGURE 12. PERFORMANCE OF A SINGLE REACTOR AT  $T = 12$  min

FIGURE 13. PERFORMANCE OF A SINGLE REACTOR AT  $T=16$  minFIGURE 14. PERFORMANCE OF A SINGLE REACTOR AT  $T=24$  min



It is evident from the plots that the average performance of the stake and stator paddle was better than that of the turbine paddle. This was found to be true for all 28 runs of this series. The statistical significance of the difference in performance was tested by the following procedure [38]. Let  $X_{T,i}$  be the individual values of the performance parameter  $n_0/n_1$  obtained in repeated runs at fixed conditions with the turbine paddle, and  $X_{S,i}$  the corresponding values obtained with the stake and stator paddle. Assuming that within each sample the values of  $X_i$  are normally distributed around their average,  $\bar{X}$ , with a standard deviation,  $S$ , it follows that the difference in the means,  $\bar{X}_d = \bar{X}_S - \bar{X}_T$ , of a pair of samples is also normally distributed with a standard deviation which is given by

$$S_d^2 = \frac{S_T^2}{N_T} + \frac{S_S^2}{N_S} \quad (94)$$

where  $S_d$  is the standard deviation of the difference,  $X_d$ , and  $N_T$  and  $S_T$  are the number of observations and standard deviation of a sample obtained with a turbine paddle.  $N_S$  and  $S_S$  are the analogous values for the stake and stator paddle.

A measure of the statistical significance of the difference,  $\bar{X}_d$  is obtained by determining the probability of obtaining a value equal to or greater than  $\bar{X}_d$  from a population normally distributed with zero average and standard deviation,  $S_d$ . This probability was calculated for each of the 28 pairs of samples and is given under the column P in Table I. The detailed calculations and the procedure for determining the standard deviation within the individual samples are given in Appendix C.

In 21 out of the 28 conditions tested (see Table I) the value of P was less than 10 percent. In two cases only was P larger than 25 percent, and under no conditions did it exceed 34 percent. The conclusion from this test is that the difference in performance between the two types of paddles is statistically significant, and that the stake and stator paddle arrangement performs better than the turbine paddle.

There are two ways through which the agitation device may affect the flocculation performance parameter. One is the intensity and structure of the turbulence field expressed by  $K_p$  and  $K_s$ . The other is the residence time distribution. The degree of completion of a chemical reaction of any order other than zero depends on the residence time distribution of the reactor in which the reaction takes place [39]. Since flocculation is not a zero order reaction it is reasonable to expect an effect of residence time distribution on performance. This was in fact proved experimentally by Harris, Kaufman, and Krone [20]. As the current study is primarily concerned with the effect of turbulence on flocculation kinetics, it was necessary to establish the relationship between the power input and the residence time distribution for the two stirring devices. The experimental procedure and the results of the residence time distribution studies are the subject of the following section.

TABLE I

STATISTICAL SIGNIFICANCE OF DIFFERENCE IN PERFORMANCE  
WITH DIFFERENT PADDLES

| Run No. | G<br>sec <sup>-1</sup> | T<br>min | $\bar{X}_S$ | $\bar{X}_T$ | $X_d/S_d$ | P<br>percent |
|---------|------------------------|----------|-------------|-------------|-----------|--------------|
| 1       | 30                     | 8        | 1.98        | 1.74        | 2.11      | 1.74         |
| 2       | 45                     | 8        | 2.10        | 1.91        | 1.38      | 8.32         |
| 3       | 60                     | 8        | 2.26        | 2.07        | 1.27      | 10.20        |
| 4       | 90                     | 8        | 2.57        | 2.24        | 1.54      | 6.18         |
| 5       | 120                    | 8        | 2.52        | 2.36        | 0.87      | 19.22        |
| 6       | 180                    | 8        | 2.25        | 1.84        | 3.00      | 0.13         |
| 7       | 240                    | 8        | 1.80        | 1.61        | 1.67      | 5.05         |
| 8       | 30                     | 12       | 2.12        | 2.00        | 0.67      | 25.14        |
| 9       | 45                     | 12       | 2.37        | 2.22        | 1.03      | 15.15        |
| 10      | 60                     | 12       | 2.65        | 2.12        | 2.45      | 0.71         |
| 11      | 90                     | 12       | 2.58        | 2.38        | 0.92      | 17.88        |
| 12      | 120                    | 12       | 2.27        | 2.18        | 0.47      | 31.92        |
| 13      | 180                    | 12       | 2.06        | 1.74        | 1.78      | 3.75         |
| 14      | 240                    | 12       | 1.69        | 1.50        | 1.27      | 10.20        |
| 15      | 30                     | 16       | 2.37        | 1.90        | 3.24      | 0.06         |
| 16      | 45                     | 16       | 2.52        | 2.09        | 2.66      | 0.39         |
| 17      | 60                     | 16       | 2.48        | 2.29        | 0.90      | 18.41        |
| 18      | 90                     | 16       | 3.14        | 2.51        | 2.44      | 2.73         |
| 19      | 120                    | 16       | 2.75        | 2.03        | 3.12      | 0.09         |
| 20      | 180                    | 16       | 2.54        | 1.79        | 3.29      | 0.05         |
| 21      | 240                    | 16       | 2.06        | 1.52        | 3.09      | 0.10         |
| 22      | 30                     | 24       | 2.75        | 2.35        | 2.55      | 0.54         |
| 23      | 45                     | 24       | 2.84        | 2.54        | 1.61      | 5.37         |
| 24      | 60                     | 24       | 3.00        | 2.72        | 1.60      | 5.48         |
| 25      | 90                     | 24       | 3.06        | 2.52        | 3.42      | 0.03         |
| 26      | 120                    | 24       | 2.84        | 2.12        | 3.85      | 0.01         |
| 27      | 180                    | 24       | 2.13        | 2.07        | 0.42      | 33.72        |
| 28      | 240                    | 24       | 2.04        | 1.47        | 3.35      | 0.04         |

The second series of flocculation tests was devoted to studying the effect of compartmentalization on flocculation performance. Two and four compartment systems were studied with the stake and stator stirrer. The results are shown in Figures 15 through 18 and in Appendix B. For comparison, single compartment data are also shown in these figures. The increase in performance with degree of compartmentalization is apparent.

#### RESIDENCE TIME DISTRIBUTION IN STIRRED REACTORS

The objective of the residence time distribution study was twofold. The first series of runs was designed to determine the effect of the total energy input and the mean residence time on the residence time distribution with each stirring device. The second series was aimed at determining the effect of paddle type on the distribution.

Residence time distribution curves were experimentally obtained from tracing tests with Pontacyl Brilliant Pink B. A transient of tracer was introduced with a syringe needle into the water stream entering the reactor. The concentration of tracer in the effluent at various times after injection was determined with a G. K. Turner Associates Model III fluorometer, using the procedure of Feuerstein and Selleck [40].

The effect of power input on the residence time distribution is shown in Figure 19. The three curves were taken at a fixed mean residence time of 8 minutes with varying  $G$  from 30 to 240  $\text{sec}^{-1}$ . The ordinates in this figure are proportional to the absolute concentration of the tracer which depend on the amount of tracer injected. Variation in the ordinates of the three curves may be attributed to variations in the amount of tracer used. Other than these minor differences the three curves are essentially the same, meaning that the power input had no effect on the residence time distribution. The same is true with regard to the effect of mean residence time as can be seen from Figure 20. Figures 21 and 22 show the effect of paddle type on the residence time distribution at three different power inputs. Here also it may be concluded that there is practically no effect.

The residence time distribution curves were further analyzed following the method of Wolf and Resnick [41]. The  $F$ -function, defined by

$$F = \frac{\int_0^{t/T} C \, dt}{\int_0^{\infty} C \, dt} \quad (95)$$

where  $C$  is the tracer concentration in the effluent, was calculated by measuring areas under the  $C$  versus  $t/T$  curve. According to Rebbur

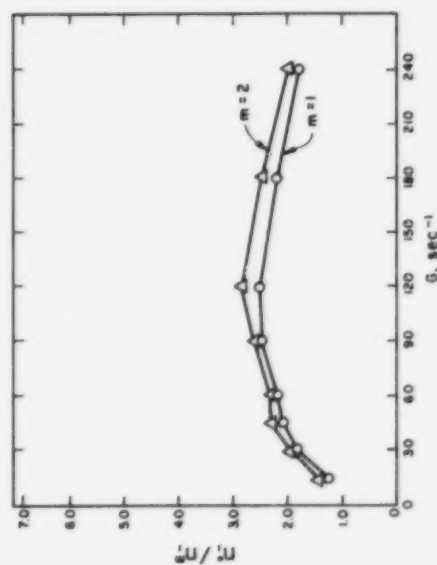


FIGURE 15. PERFORMANCE OF MULTICOMPARTMENT SYSTEMS  $T = 8$  min

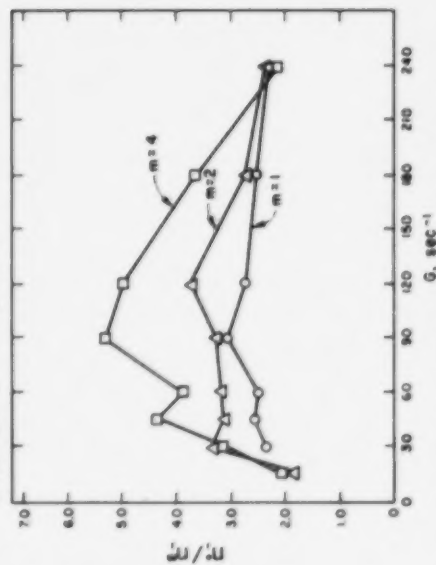


FIGURE 17. PERFORMANCE OF MULTICOMPARTMENT SYSTEMS  $T = 16$  min

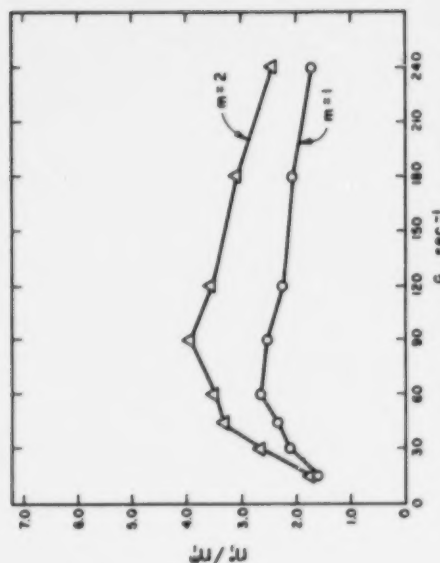


FIGURE 16. PERFORMANCE OF MULTICOMPARTMENT SYSTEMS  $T = 12$  min

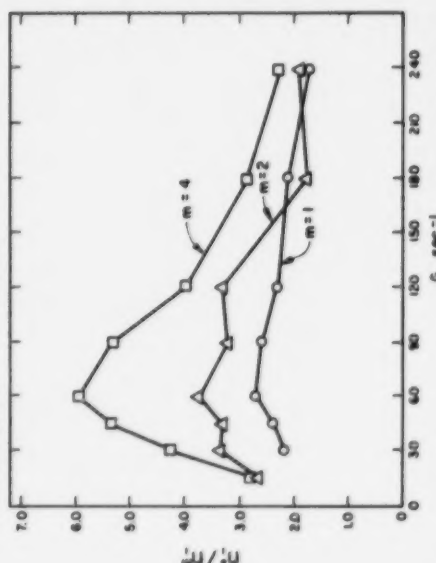


FIGURE 18. PERFORMANCE OF MULTICOMPARTMENT SYSTEMS  $T = 24$  min

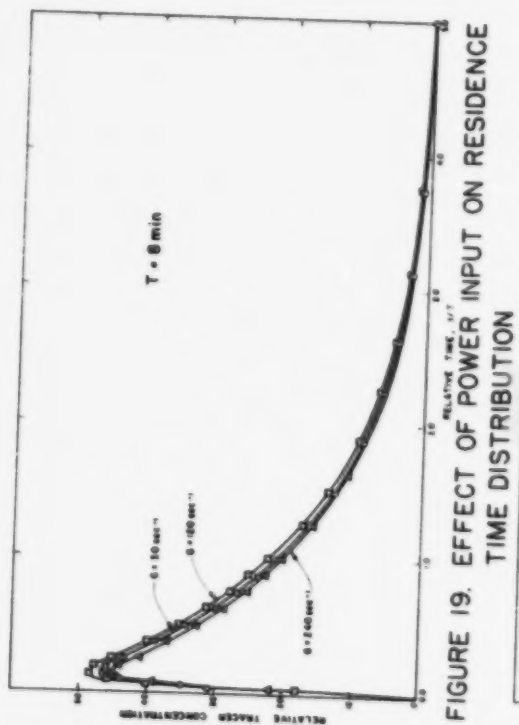


FIGURE 19. EFFECT OF POWER INPUT ON RESIDENCE TIME DISTRIBUTION

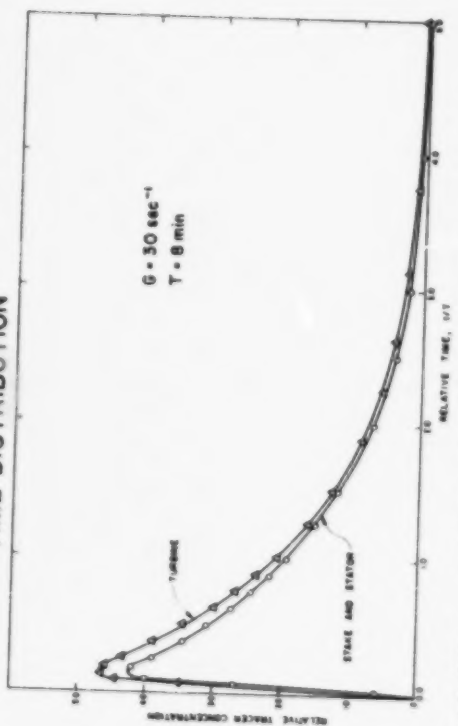


FIGURE 21. EFFECT OF PADDLE TYPE ON RESIDENCE TIME DISTRIBUTION

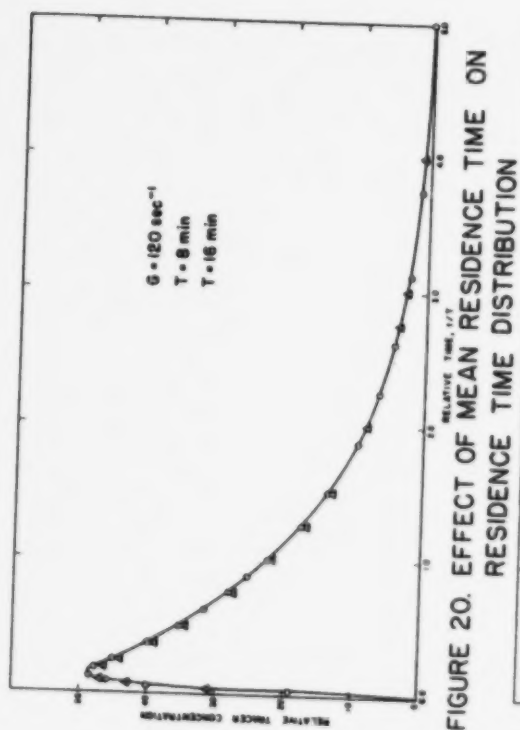


FIGURE 20. EFFECT OF MEAN RESIDENCE TIME ON RESIDENCE TIME DISTRIBUTION

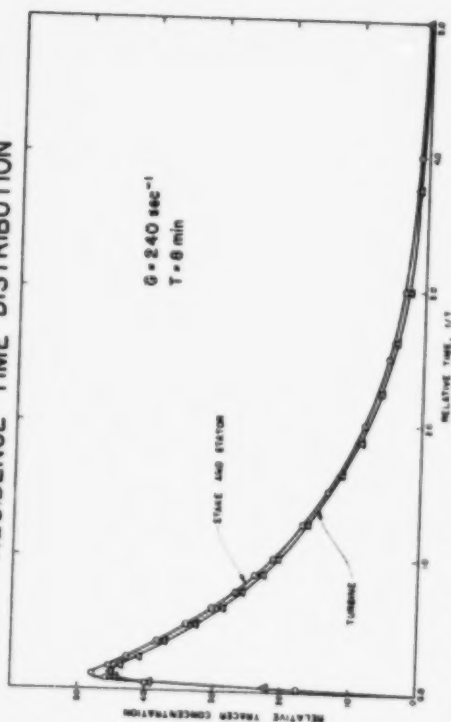


FIGURE 22. EFFECT OF PADDLE TYPE ON RESIDENCE TIME DISTRIBUTION



and Arguman [42] the F-function for a hypothetical continuous flow system consisting of "dead space," "perfect mixing," and "plug flow" is given by

$$F(t) = 1 - \exp \left[ - \frac{1}{(1-p)(1-m)} \left[ \frac{t}{T} - p(1-m) \right] \right] \quad (96)$$

where  $m$  is the "dead space" fraction of the reactor volume,  $p$  is the "plug flow" fraction of the effective volume, and  $1-p$ , the "perfect mixing" fraction. The flowthrough characteristics of a real system can be characterized quantitatively by evaluating  $p$  and  $m$ . Equation 96 predicts a linear relationship between the Logarithm of  $1-F(t)$  and  $t/T$ .

Figure 23 is a plot of the experimental values of  $1-F$  versus  $t/T$  for the single compartment reactor (calculated from Figure 21, stake and stator) from which the values of  $m$  and  $p$  were found to be  $-0.005$  and  $0.073$  respectively. These results indicate that the system's flowthrough characteristics are equivalent to those of an ideal system with  $-0.5$  percent dead space and with  $92.7$  percent of its effective volume completely mixed, the remainder  $7.3$  percent being plug flow. The negative value of  $m$  has no physical meaning, and is probably due to experimental error. Moreover, since  $m$  was found to be very small it may be assumed that there is practically no dead space in the system.

#### FLOC SIZE MEASUREMENTS

In connection with the assumption made in Chapter III regarding the dependence of floc size on power input (cf., Equation 76), a series of floc size measurements was made. Three alternative methods for these measurements were considered, namely, settling, the Coulter counter technique, and direct photography. The settling test, which is relatively easy to perform, was ruled out because of possible variations in floc properties during the settling period and also because it provides information on floc settling velocity rather than physical size. The Coulter counter technique was not acceptable because floc breakup was likely to occur, either during sampling or in the measuring cell where flocs must pass through a small orifice. The direct photographic method was adopted since it enabled positive and direct measurements of flocs as they were formed in the reactor. The apparatus used for these measurements consisted of a photographic cell, a microscope, and a polaroid camera. The photographic cell was attached to the sidewall of the reactor enabling the withdrawal of samples with minimum floc disruption. The depth of sample in the photographic cell was  $1.6$  mm. The image of the flocs was projected through a microscope into a polaroid camera producing an enlarged positive picture of the flocs. The overall magnification of the optical system was determined experimentally and found to be  $18.5$ .

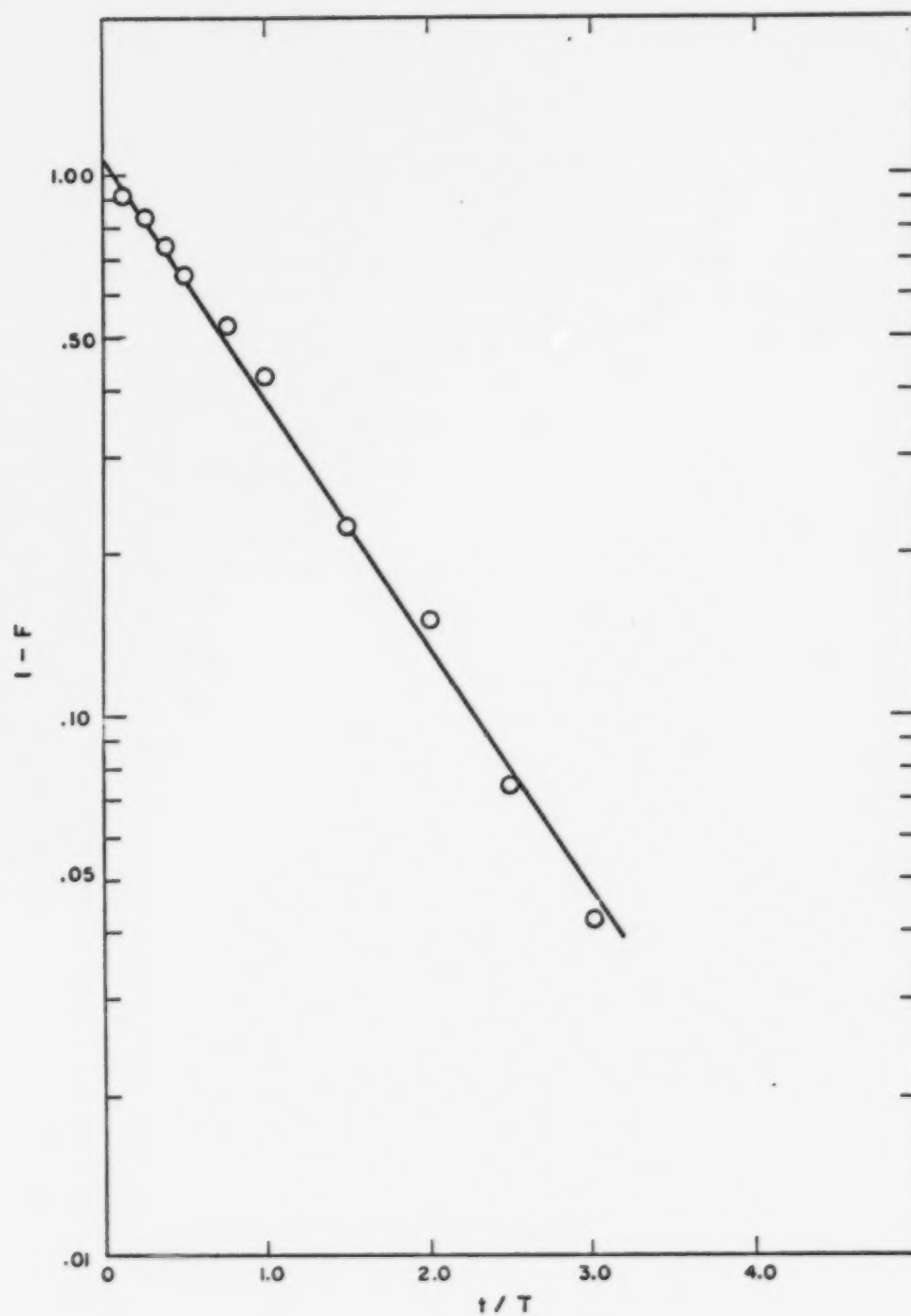


FIGURE 23. RESIDENCE TIME DISTRIBUTION FUNCTION

Floc size measurements were carried out on batch systems so that floc growth with time could be followed. The experiments were done with both types of stirring devices, and with  $G$  varying from 30 to 240  $\text{sec}^{-1}$ . Knowing the true magnification of the optical system, average floc sizes were estimated from the size of their images on the photographs. Out of the 40 pictures taken in this series, six are shown in Figures 24 and 25. Floc size was determined from these pictures by measuring the maximum and minimum diameters of at least 20 flocs picked randomly. The complete results are shown in Figures 26 and 27 from which it can be seen that in most cases the flocs reached their ultimate size after approximately six minutes. The effect of  $G$  on the ultimate floc size is shown in Figure 28. There seems to be no important effect of paddle type on floc size. A plot of the reciprocal of floc diameter versus  $G$  is shown in Figure 29. As  $u^2$  is a linear function of  $G$ , the straight-line relationship of Figure 29 indicates the general validity of Equation 76 with  $K_3 = 1.00$ , at least for the case of the test water used in these studies.

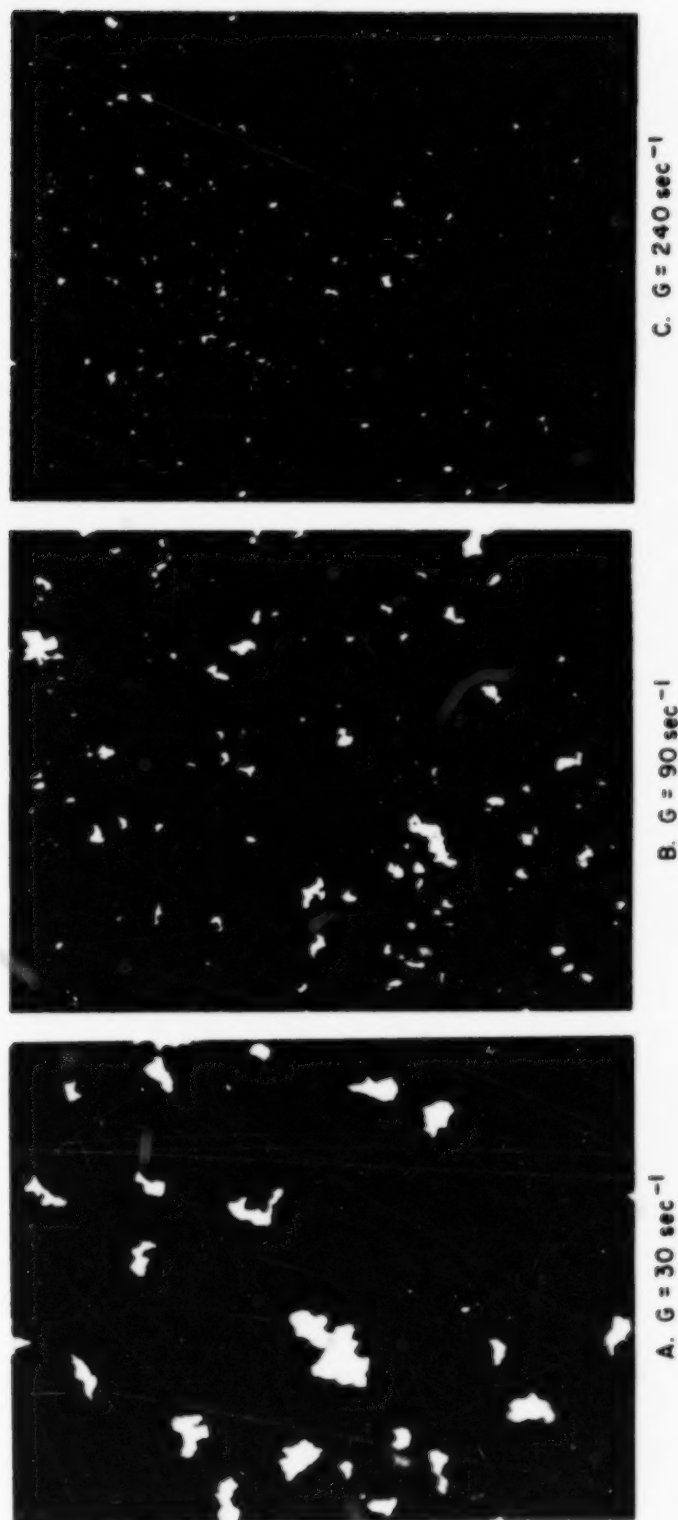
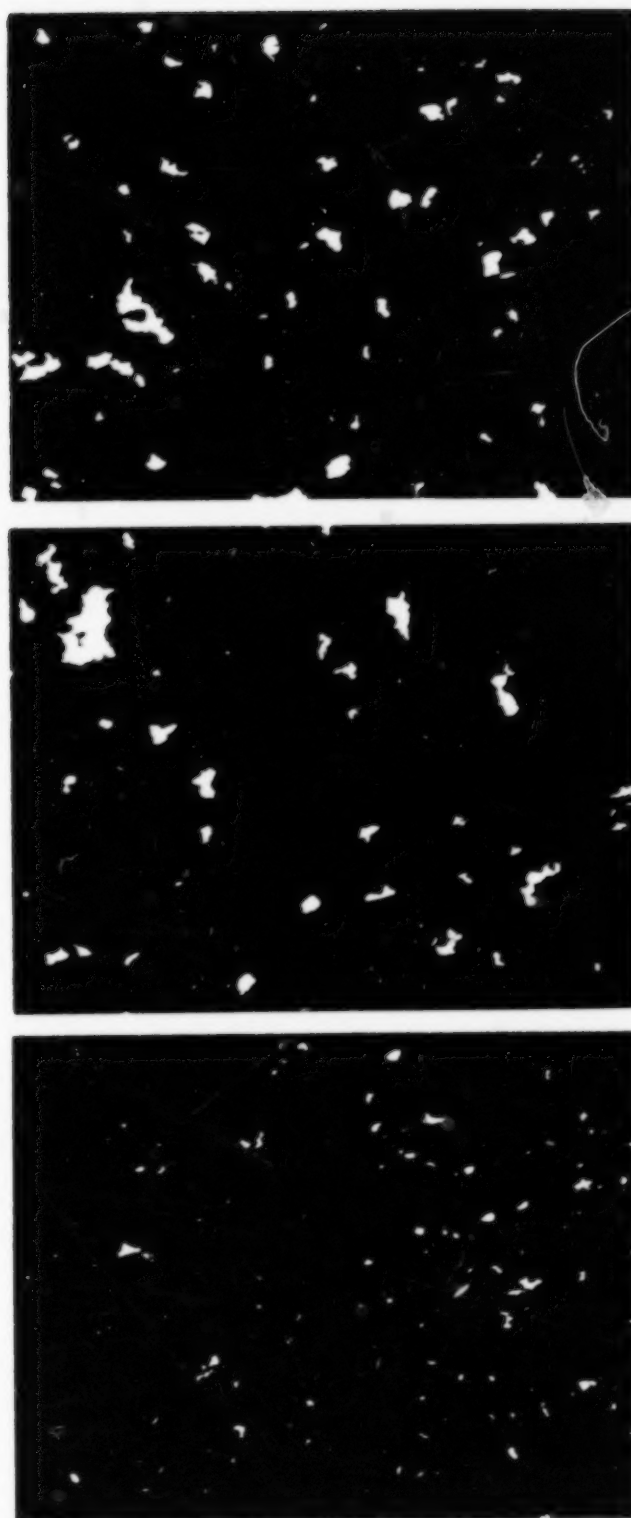


FIGURE 24. VARIATION OF FLOC SIZE WITH  $G$ . TURBINE PADDLE;  $T = 20 \text{ min}$  (BATCH SYSTEM)



A.  $T = 2 \text{ min}$

B.  $T = 6 \text{ min}$

C.  $T = 20 \text{ min}$

FIGURE 25. VARIATION OF FLOC SIZE WITH TIME. STAKE AND STATOR PADDLE;  $G = 60 \text{ sec}^{-1}$   
(BATCH SYSTEM)



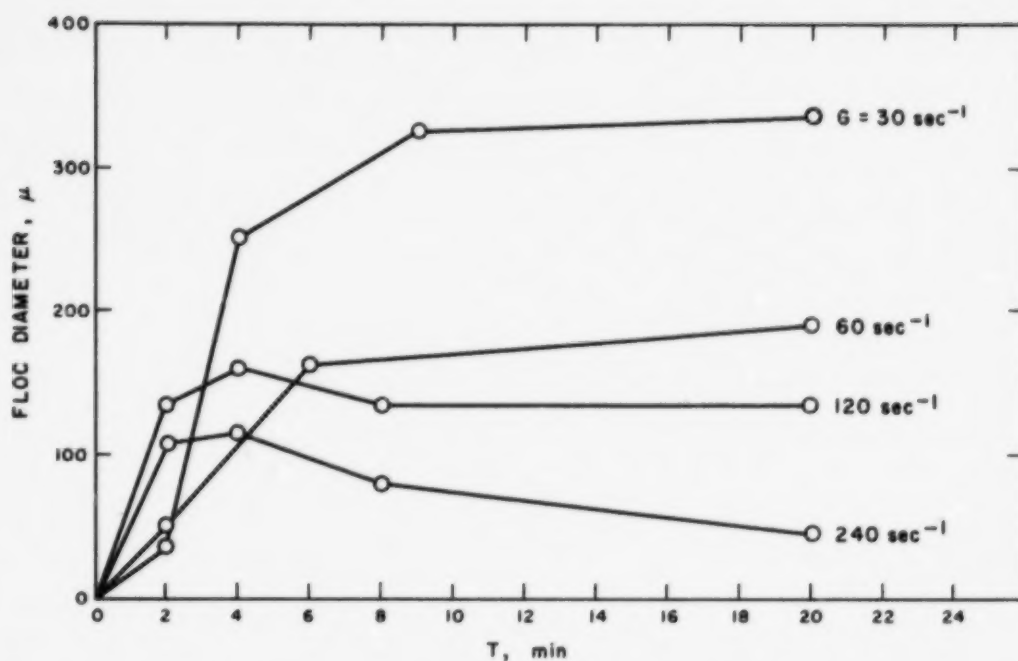


FIGURE 26. FLOC GROWTH IN BATCH SYSTEMS. STAKE AND STATOR PADDLE

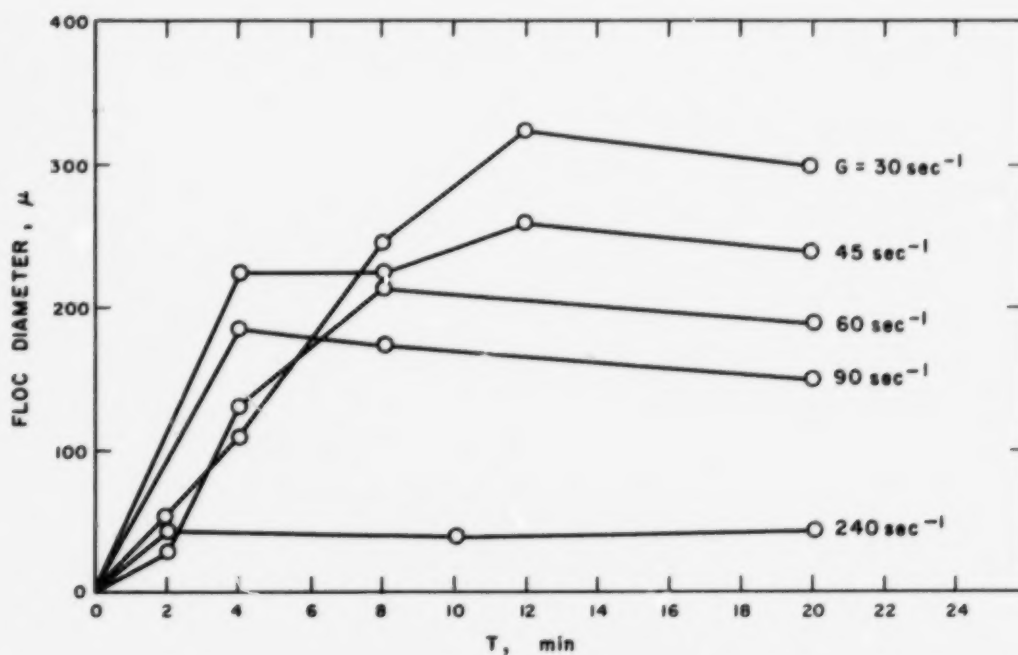


FIGURE 27. FLOC GROWTH IN BATCH SYSTEMS. TURBINE PADDLE

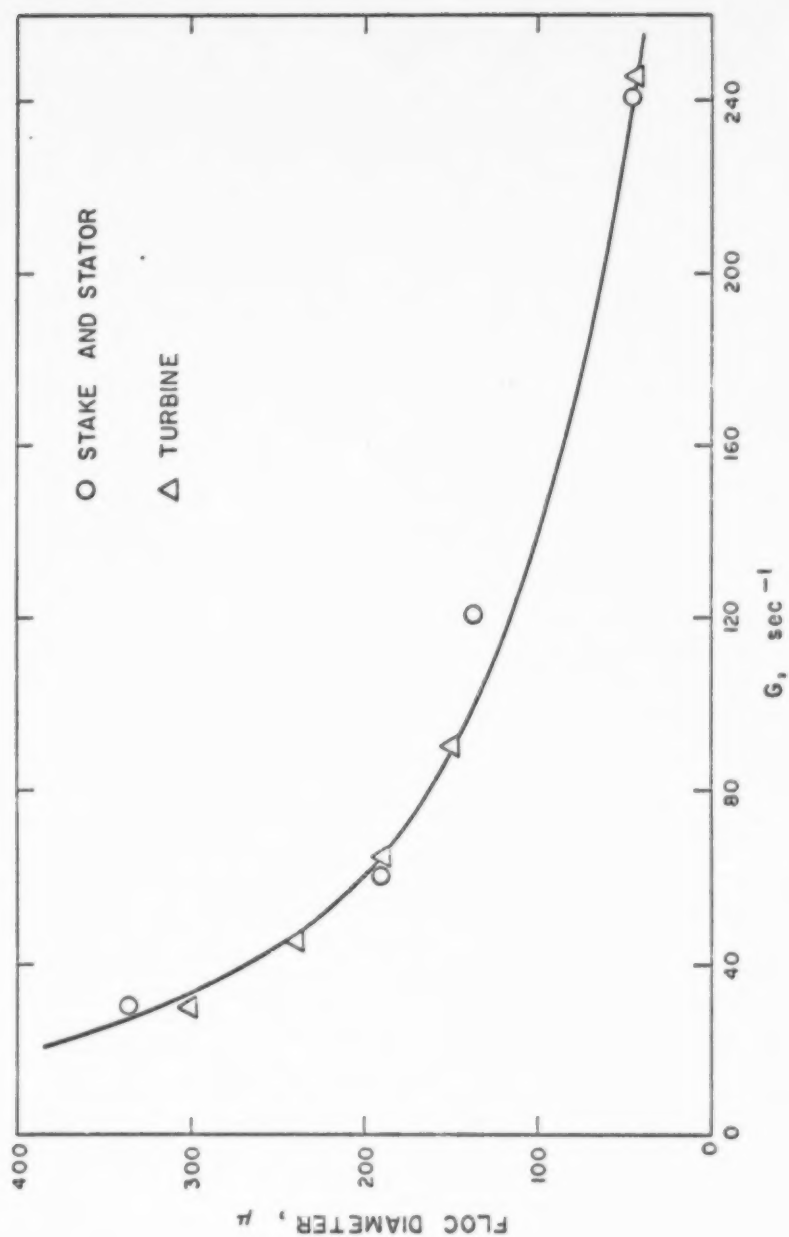


FIGURE 28. ULTIMATE FLOC SIZE IN BATCH SYSTEM

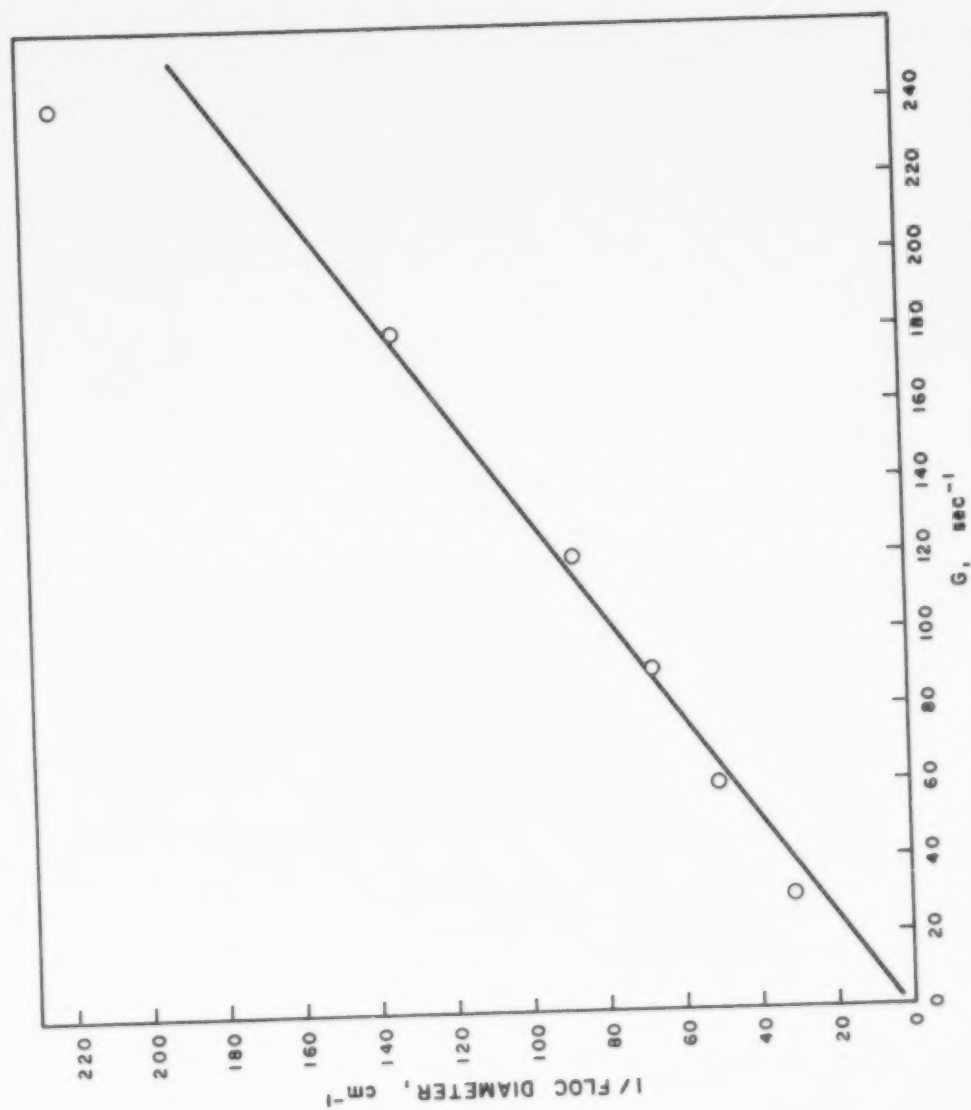


FIGURE 29. RECIPROCAL FLOC DIAMETER VERSUS  $G_t$  BATCH SYSTEM

## V. LABORATORY INVESTIGATION OF TURBULENCE IN STIRRED REACTORS

### OBJECTIVES OF TURBULENCE STUDIES

The flocculation experiments described in the previous chapter have shown that the geometry and operation of the agitating device influences the effectiveness of the system in carrying out the flocculation process. With equal power inputs the stake and stator paddle arrangement gave consistently better results than the turbine type paddle. Tracing tests have shown that with equal energy inputs the residence time distribution with one paddle type was essentially the same as with the other, indicating that this could not account for the difference in performance. The theoretical model for turbulent flocculation presented in Chapters II and III suggests that the structure of the turbulence field, i.e., the energy spectrum, may affect the rate of collisions and the breakup thus influencing the kinetics of the flocculation process.

With these experimental results and theoretical concepts as a basis it seemed appropriate and necessary to continue the study by an experimental investigation of the turbulence field created by the two agitating devices. The experimental work was aimed at determining the spatial distribution of kinetic energies in the reactor with each of the stirring systems and the variations in energy spectrums between the two.

### THE HOT-FILM ANEMOMETER

The hot-film anemometer system used in this study was the Model 1010A Constant Temperature Anemometer, product of Thermo-Systems, Inc. The system was designed to maintain a resistance element (sensor) at a fixed temperature. The sensor was exposed directly to the fluid whose velocity was to be measured. The variation in the electric current required to maintain the sensor at a constant temperature was then related to the changes in velocity past the sensor. The sensor consisted of a glass fiber with a thin platinum coating protected by a sputtered quartz film. The sensing element was 0.001 in. in diameter and approximately 0.02 in. long. It was mounted on a sensor holder and connected to the electronic circuitry through a probe as shown in Figure 30.

The basic equation relating the fluid velocity past the sensor to the heat dissipated in the sensor is known as King's equation [22] and is given by

$$q = A' + B'U_{\text{eff}}^{\frac{1}{n}} \quad (97)$$

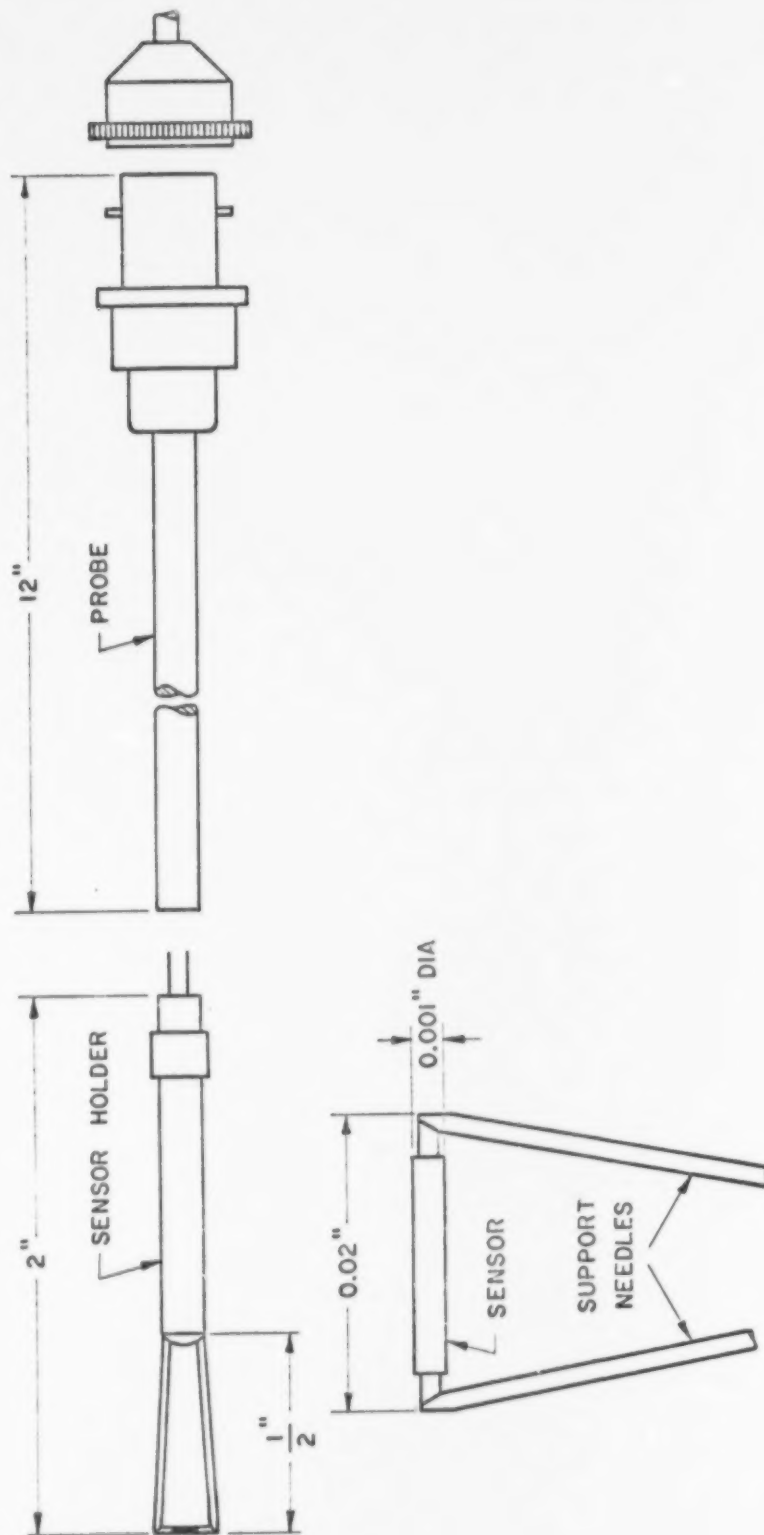


FIGURE 30. HOT-FILM ANEMOMETER SENSOR AND PROBE



where  $q$  is the power supplied to the sensor,  $U_{eff}$  is the effective velocity past the sensor defined in Figure 31,  $A'$ ,  $B'$ , and  $n$  are constants depending on the properties of the sensor and the fluid.

The basic output signal of the system is the voltage across a bridge which is proportional to the current through the sensor. Since the sensor's resistance is maintained at a constant level, this bridge voltage is proportional to the power dissipated in the sensor. Thus, from Equation 97, one obtains

$$E_b^2 = A + B U_{eff}^{\frac{1}{n}} \quad (98)$$

where  $E_b$  is the bridge voltage and  $A$ ,  $B$ , and  $n$  are constants.

The constants of Equation 98 were determined experimentally by inserting the probe into a 3/4-in. diameter pipe through which water was pumped at known flow rates. The value of the constant  $A$  was simply the square of the bridge voltage at zero velocity.  $B$  and  $n$  were then calculated from a plot of  $(E_b^2 - A)$  versus  $U_{eff}$ , shown in Figure 32. The values of the constants thus determined were:

$$A = 24.9 \quad B = 18.0 \quad \text{and} \quad n = 2.08$$

The theoretical value of  $n$  is 2.0 which is reasonably close to the experimental value.

Inserting the values of the constants into Equation 98 and rearranging yields the working equation for the hot-film anemometer

$$U_{eff} = \left( \frac{E_b^2 - 24.9}{18.0} \right)^{2.08} \quad (99)$$

where the effective velocity is in cm/sec and the bridge voltage in volts. Using a test procedure suggested by the manufacturer, the frequency response of the system was found to be on the order of 2000 cycles per second.

#### DETERMINING THE MEAN SQUARE VELOCITY FLUCTUATIONS

A measure of the intensity of turbulence is the mean square of the velocity fluctuations. This quantity can be determined experimentally with the hot-film anemometer and a root-mean-square voltmeter [43]. When a sensor is placed in a turbulent fluid with its axis perpendicular to the direction of mean flow, the effective velocity past the sensor, according to Figure 31, is given by

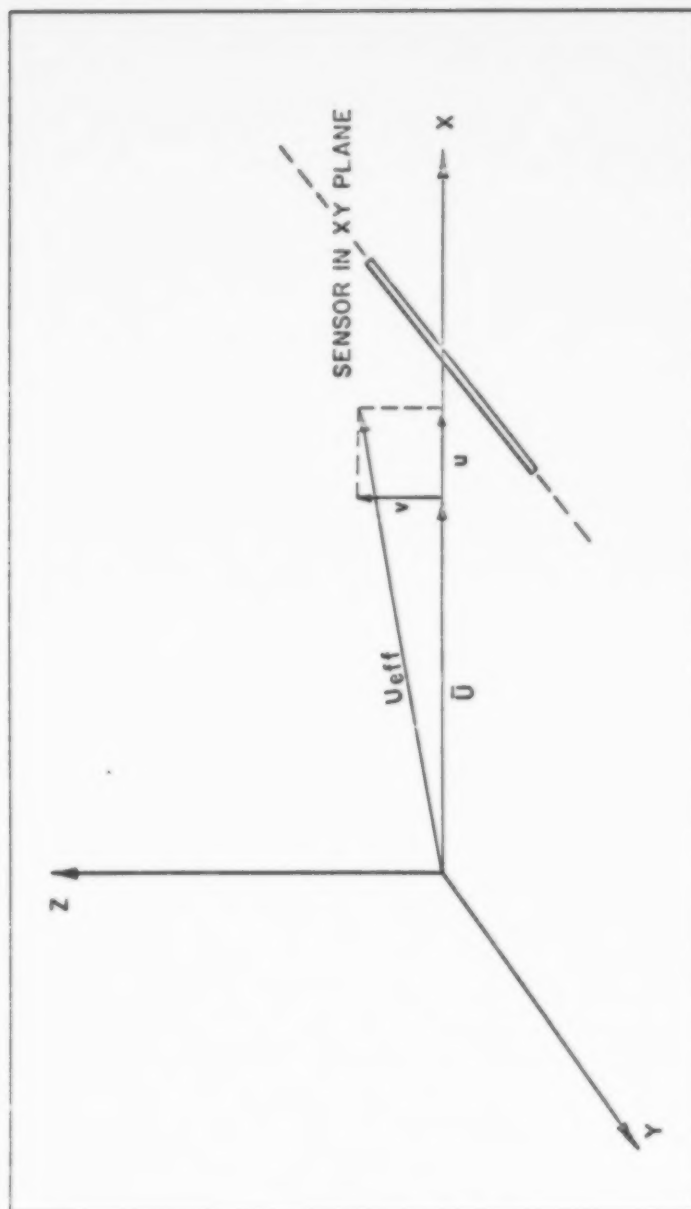


FIGURE 31. RELATION BETWEEN MEAN AND EFFECTIVE VELOCITIES

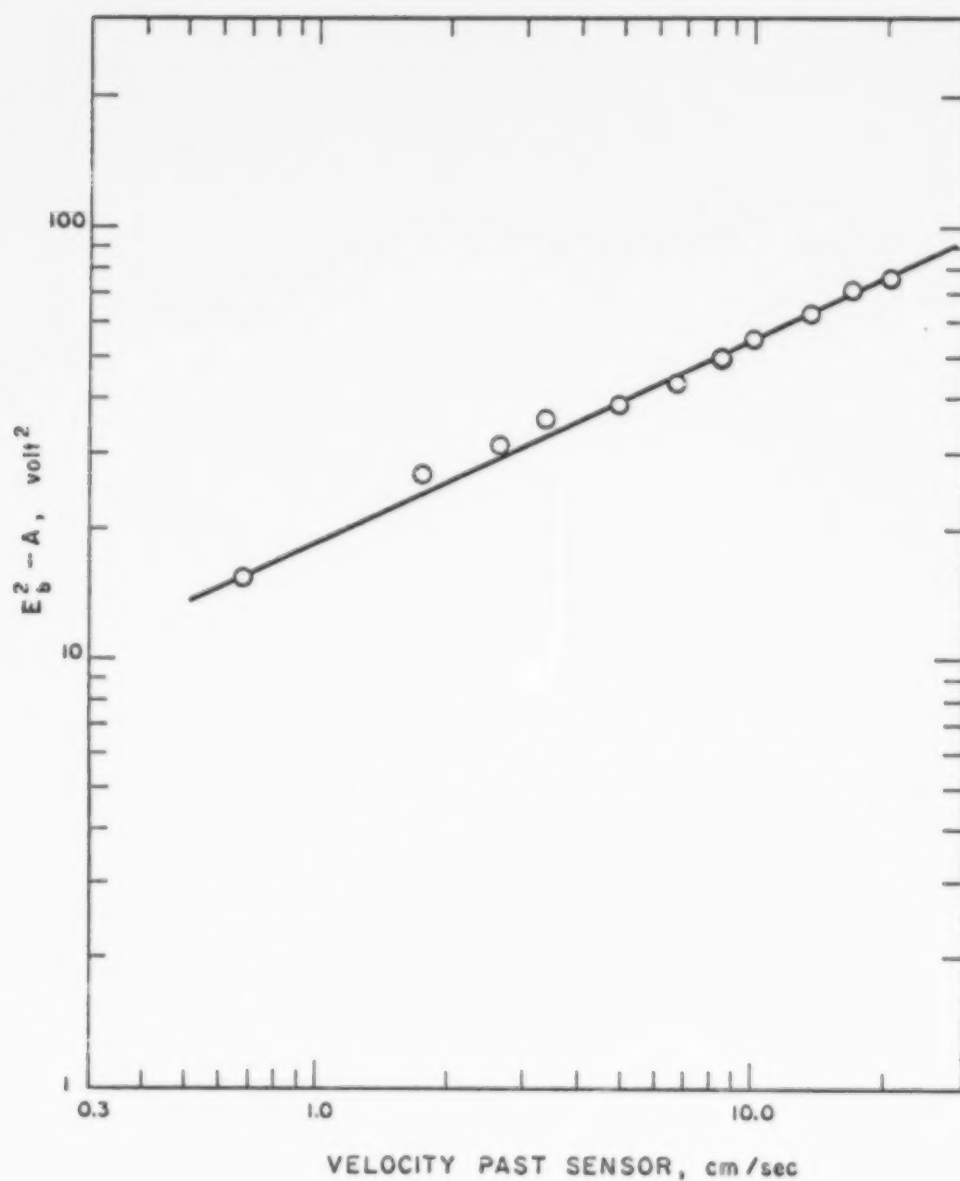


FIGURE 32. CALIBRATION OF HOT-FILM ANEMOMETER

$$U_{\text{eff}} = [(\bar{U} + u)^2 + v^2]^{\frac{1}{2}} \quad (100)$$

where  $\bar{U}$  is the mean velocity and  $u$  and  $v$  are the fluctuating velocities perpendicular to the sensor's axis. The sensor is assumed to be relatively insensitive to velocities (or components) parallel to its axis. Since  $u$  and  $v$  vary with time, only an average value for  $U_{\text{eff}}$  can be measured. Expanding Equation 100 and averaging each term results in

$$\bar{U}_{\text{eff}} = \bar{U} \left[ 1 + \frac{\overline{u^2}}{\bar{U}^2} + \frac{\overline{v^2}}{\bar{U}^2} \right]^{\frac{1}{2}} \quad (101)$$

The contribution of the turbulent fluctuations to the average effective velocity is negligible when the "relative turbulence intensities"

$\sqrt{\overline{u^2}}/\bar{U}$  and  $\sqrt{\overline{v^2}}/\bar{U}$  are small. For relative turbulence intensity of 0.10, an error of one percent is made by neglecting the turbulence velocity fluctuation. The error is 13 percent for relative intensity of 0.50.

Differentiating Equation 98 yields

$$2 \bar{E}_b d\bar{E}_b = \frac{1}{n} B U_{\text{eff}}^{1/n-1} dU_{\text{eff}} \quad (102)$$

Combining Equations 98 and 102,

$$\frac{2 \bar{E}_b d\bar{E}_b}{\bar{E}_b^2 - A} = \frac{1}{n} \frac{dU_{\text{eff}}}{U_{\text{eff}}} \quad (103)$$

and taking  $d\bar{E}_b$  as a fluctuation in bridge voltage ( $e$ ) and  $dU_{\text{eff}}$  as a velocity fluctuation ( $u$ ), the relative turbulence intensity in the direction of the mean flow becomes

$$\frac{\sqrt{\overline{u^2}}}{\bar{U}} = \frac{2 n \bar{E}_b \sqrt{\overline{e^2}}}{\bar{E}_b^2 - A} \quad (104)$$

where  $\sqrt{\overline{e^2}}$  is the root-mean-square (rms) bridge voltage fluctuation.

The rms bridge voltage fluctuation was measured by the Model 3400 Hewlett Packard rms voltmeter. The frequency range of this instrument, as reported by the manufacturer, was from 2 cps to 10 Mc. The accuracy is  $\pm 5$  percent for frequencies higher than 10 cps, and

± 25 percent for frequencies between 2 cps and 10 cps. The instrument does not respond to frequencies below 2 cps.

Values of the relative turbulence intensity were determined using Equation 104. The effective velocities ( $U_{eff}$ ) were calculated from Equation 99 and the true mean velocities ( $\bar{U}$ ) from Equation 101. For these calculations, it was assumed that the turbulence was isotropic, i.e.,  $\overline{u^2} = \overline{v^2}$ .

#### HOMOGENEITY OF TURBULENCE FIELD

The homogeneity of the turbulence field was tested by measuring the mean square velocity fluctuations at nine different locations in the reactor. The probe was introduced through the bottom of the reactor at points A, B, and C, shown in Figure 33. At each of these points turbulence characteristics were measured in three different elevations: Mid-depth, 3-in. above mid-depth, and 3-in. below mid-depth. The sensor was always in a horizontal plane and oriented with its axis pointing towards the center of the reactor. This orientation of the sensor gave the largest velocities, indicating that the horizontal motion of the fluid was normal to the stirrer blades. The sensor could not be rotated in a vertical plane. The mean velocities measured in this manner are the vector sums of the horizontal tangential velocities and the vertical velocities. The direction of the mean velocity is unknown, and is of no concern.

The results of these experiments are tabulated in Appendix D and shown in Figures 34 through 37. Due to the inaccuracy of the rms voltmeter at low frequencies, the values of the mean square velocity fluctuations cannot be regarded as absolutely correct. This is particularly true for the low G runs where the contribution of low frequencies to the total  $u^2$  is presumably more significant. However, for the analysis of the homogeneity of the turbulence field, relative values rather than absolute are of interest.

The results indicate that the turbulence field is by no means homogeneous. Spatial variations of both the mean velocity and mean square velocity fluctuations are significant. There is no indication, however, that the homogeneity depends on G. Neither does it depend on the stirring device. This latter result was somewhat surprising since intuitively one would expect better homogeneity with the stake and stator paddle arrangement. For the sake of comparison, the results of the experiments are summarized in Table II where the average values of  $u^2$  and  $\bar{U}$  as well as their ranges are given for both stirring devices.

In connection with the theoretical analysis presented in Chapter III, it was particularly interesting to investigate the relation between  $u^2$  and G. The results of this are shown in Figures 38 through 40, and indicate that the assumption expressed in Equation 84 regarding the linear relationship between  $u^2$  and G is a reasonable one. But the proportionality constant,  $K_p$ , which is the paddle performance coefficient, is not constant throughout the reactor. In order to arrive at a single value for  $K_p$ , average values of  $u^2$  were calculated.



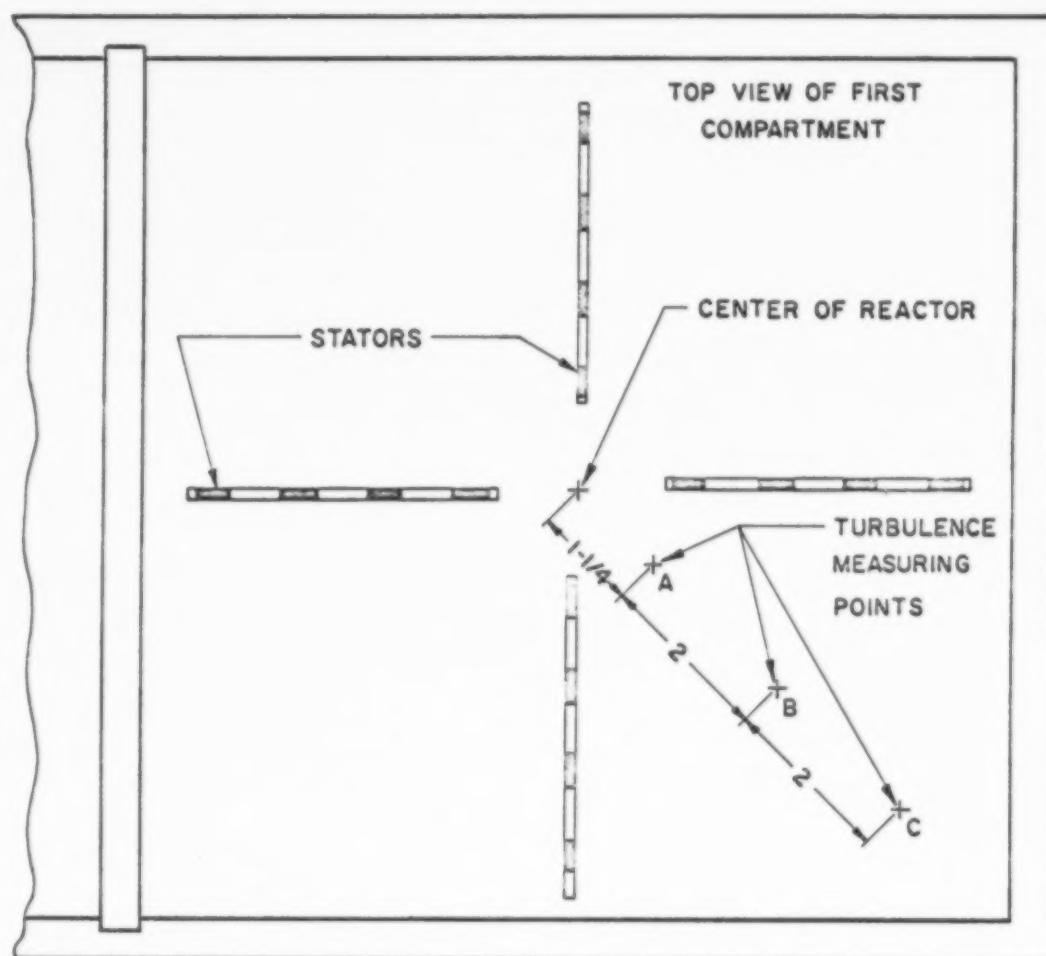


FIGURE 33. TURBULENCE MEASURING LOCATIONS

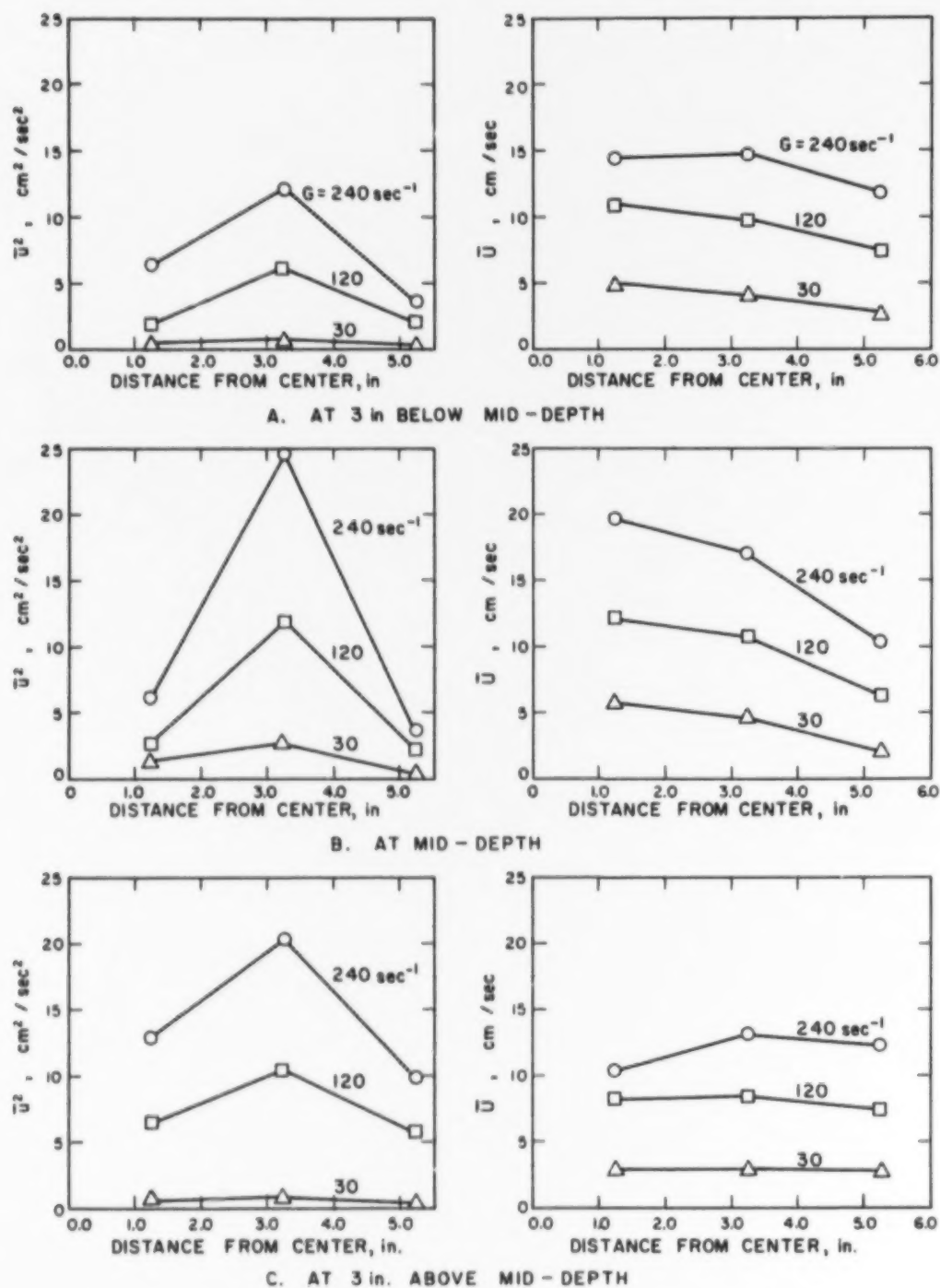


FIGURE 34. HORIZONTAL VARIATION OF TURBULENCE PROPERTIES.  
STAKE AND STATOR PADDLE

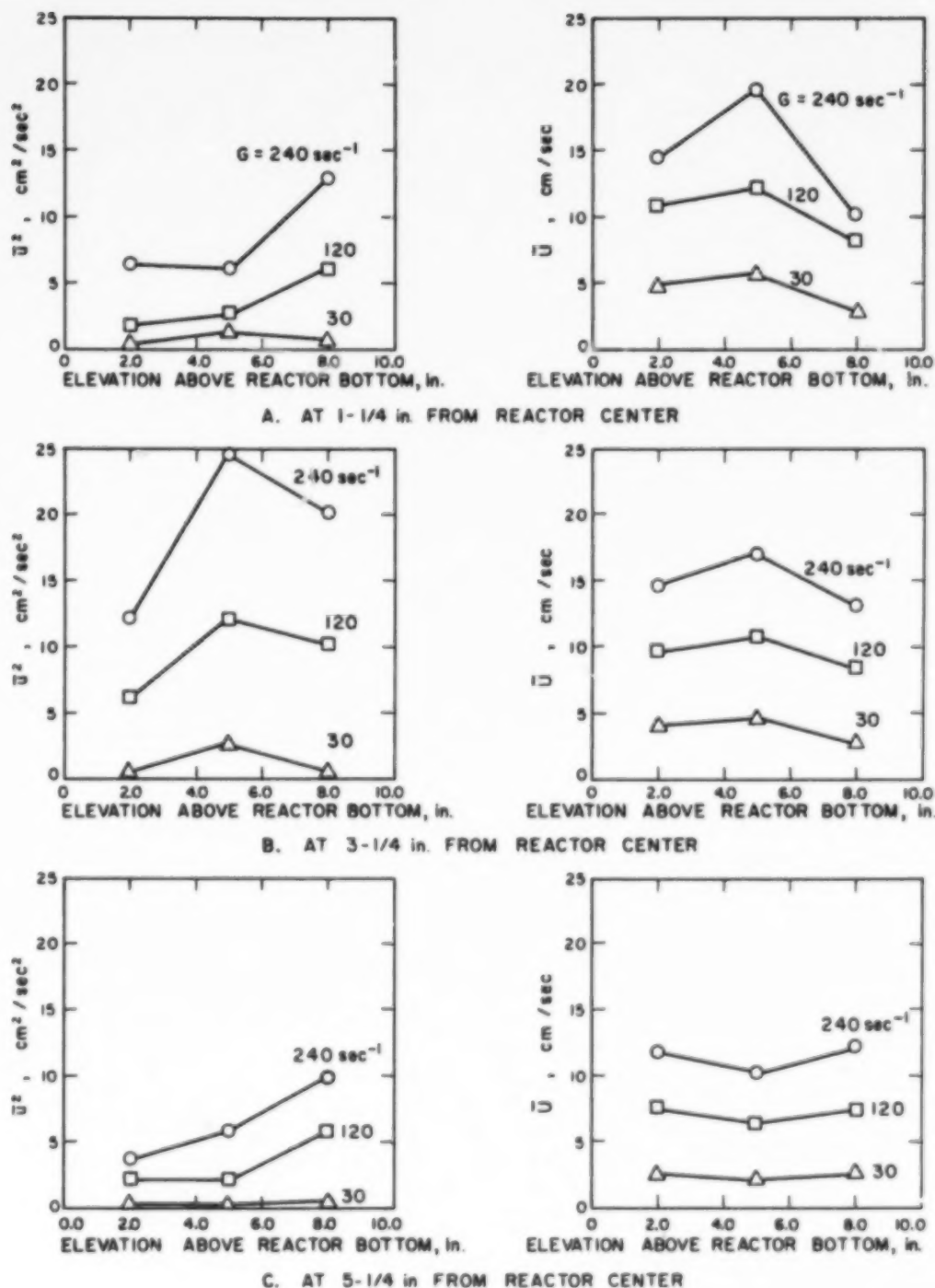


FIGURE 35. VERTICAL VARIATION OF TURBULENCE PROPERTIES  
STAKE AND STATOR PADDLE

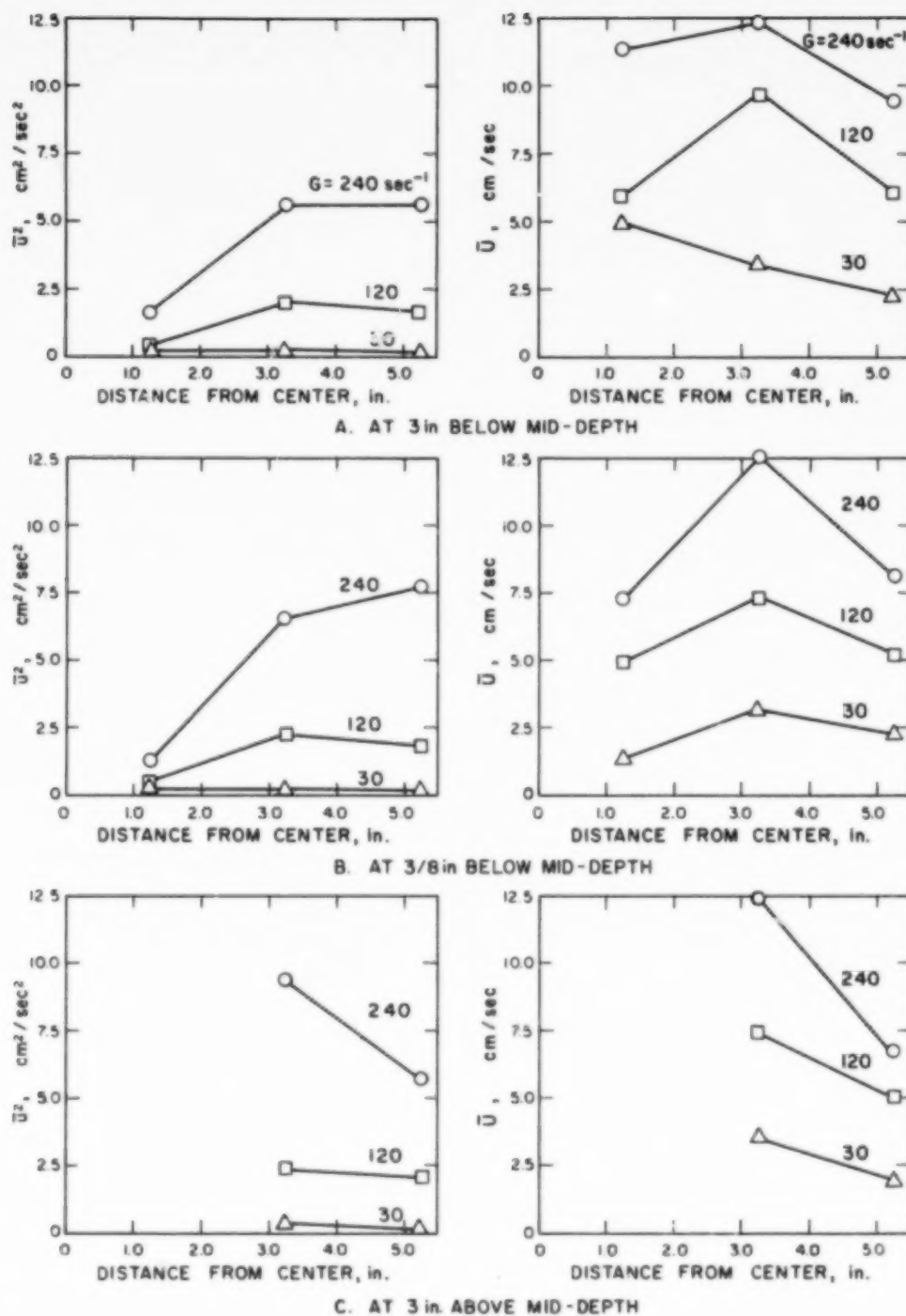


FIGURE 36. HORIZONTAL VARIATION OF TURBULENCE PROPERTIES  
TURBINE PADDLE

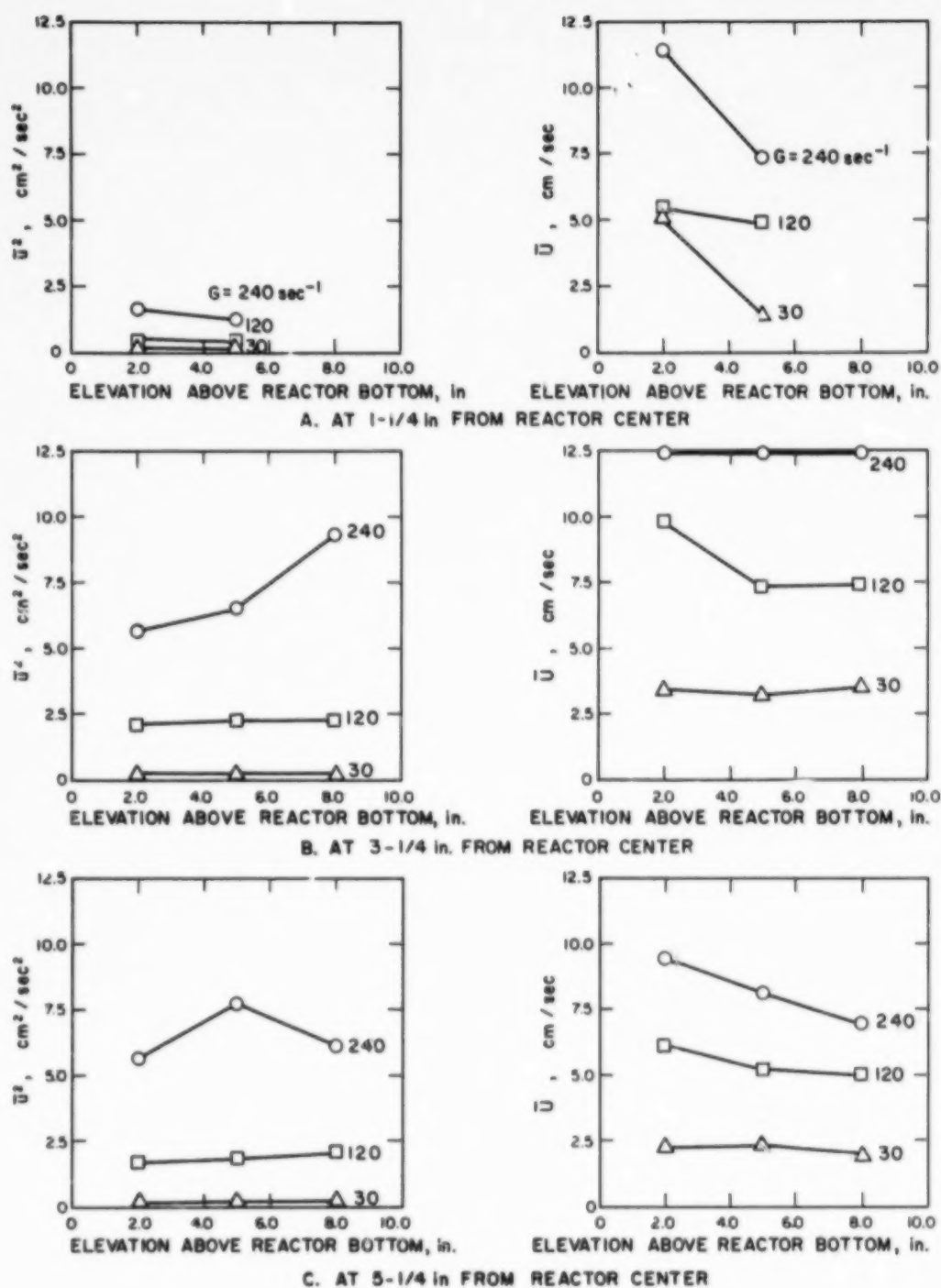


FIGURE 37. VERTICAL VARIATION OF TURBULENCE PROPERTIES  
TURBINE PADDLE



TABLE II  
SPATIAL VARIATIONS OF TURBULENCE PROPERTIES

| G<br>sec <sup>-1</sup> | Stake and Stator Paddle |             |                                                      | Turbine Paddle     |         |                                                      |       |              |
|------------------------|-------------------------|-------------|------------------------------------------------------|--------------------|---------|------------------------------------------------------|-------|--------------|
|                        | $\bar{U}$ , cm/sec      |             | $\overline{u^2}$ , cm <sup>2</sup> /sec <sup>2</sup> | $\bar{U}$ , cm/sec |         | $\overline{u^2}$ , cm <sup>2</sup> /sec <sup>2</sup> | Range |              |
|                        | Average                 | Range       | Average                                              | Range              | Average |                                                      |       |              |
| 30                     | 3.52                    | 2.1 to 5.7  | 0.93                                                 | 0.16 to 2.75       | 2.86    | 1.40 to 5.00                                         | 0.12  | 0.02 to 0.25 |
| 120                    | 8.82                    | 6.1 to 12.2 | 5.76                                                 | 1.89 to 12.20      | 6.39    | 4.9 to 9.8                                           | 1.65  | 0.42 to 2.28 |
| 240                    | 13.31                   | 9.3 to 17.1 | 12.34                                                | 3.67 to 28.50      | 10.5    | 6.7 to 13.9                                          | 5.50  | 1.26 to 9.38 |

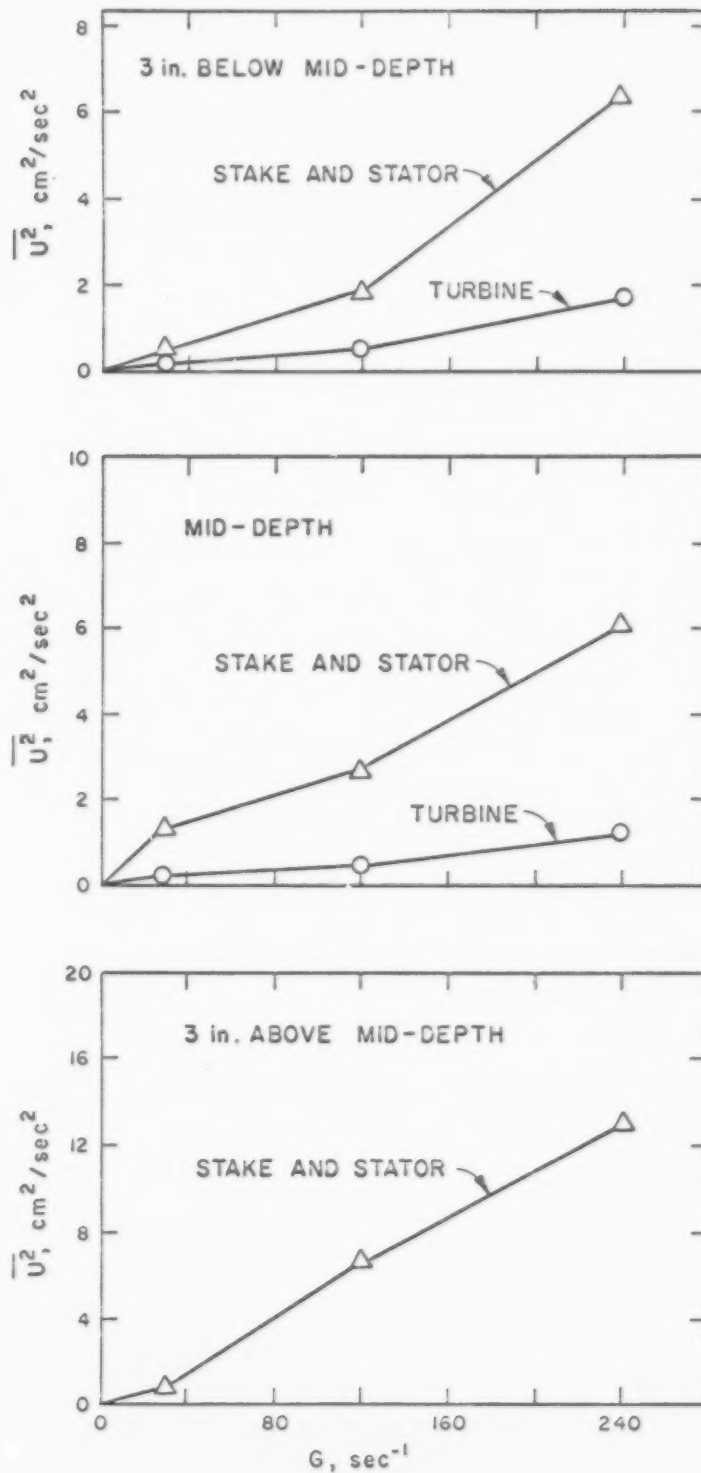


FIGURE 3C.  $\overline{U^2}$  VERSUS  $G$  AT 1-1/4 in. FROM CENTER

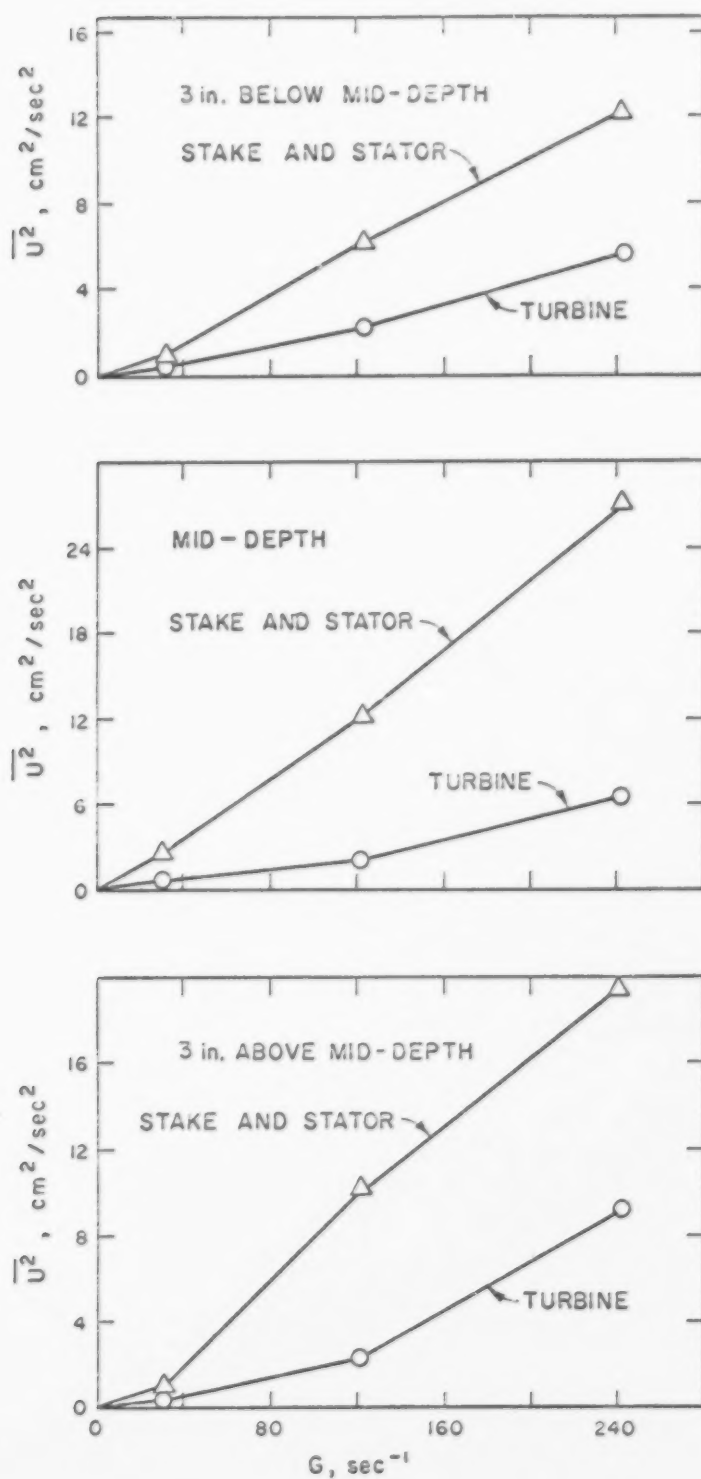


FIGURE 39.  $\overline{U^2}$  VERSUS  $G$  AT 3-1/4 in. FROM CENTER

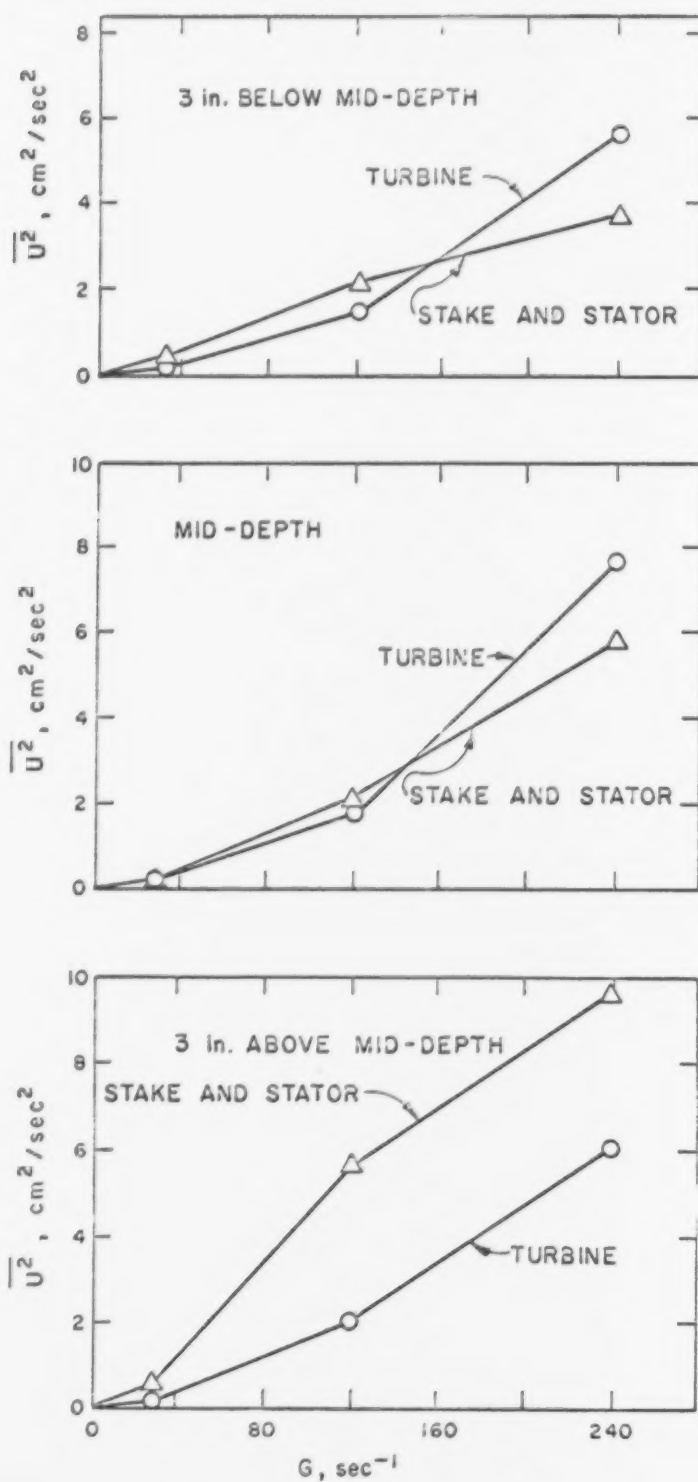


FIGURE 40.  $\overline{U^2}$  VERSUS  $G$  AT 5-1/4 in. FROM CENTER

In the averaging process each observation was assigned a weight proportional to the fluid volume it represents. This volume is proportional to the distance from the observation point to the center of the reactor. As the observation points, A, B, and C were located 1.25-in., 3.25-in., and 5.25-in. from the center, their weight factors were 0.13, 0.33, and 0.54 respectively. The calculations of the weighted averages of  $u^2$  are given in Appendix E, and the results are shown in Figure 41. From the slopes of the straight-lines in Figure 41, the paddle performance coefficients ( $K_p$ ) were determined as 0.0458 cm<sup>2</sup>/sec and 0.0233 cm<sup>2</sup>/sec for the stake and stator and turbine paddles, respectively.

#### MEASUREMENT OF ENERGY SPECTRA

The hot-film anemometer in combination with the rms voltmeter was used to estimate the mean square velocity fluctuations at various locations in the reactor under different stirring conditions. Apart from being insensitive to low frequencies, the rms voltmeter does not provide information on the energy spectrum of the turbulence, a characteristic which is believed to be of considerable significance in the kinetics of flocculation. Hence, a series of experiments aimed at determining the energy spectrum of the turbulence was carried out and is described in this section. The theoretical principles involved in these measurements were discussed in detail by Blackman and Tukey [44].

The apparatus used for spectrum measurements is shown schematically in Figure 42. It consisted of the hot-film anemometer system and a digital tape recorder. The variation of fluid velocity with time at a fixed point was recorded, and the data were then processed on the IBM 7094 computer. A similar method was used by Raichlen [45] for turbulence measurements in a water flume. Fluid velocities were measured with the hot-film anemometer as described previously. The output signal from the anemometer circuitry, i.e., the bridge voltage, was fed into the high speed digital recorder designed and built by the Hydraulic Laboratory staff at the University of California, Berkeley. The instrument was capable of recording analog signals in digital form on a magnetic tape which could subsequently be processed by the computer. The maximum sampling rate, i.e., the number of data points recorded per unit time, was 7,500 samples per second, which corresponds to a frequency response of 3,750 cps. The input into the recorder was limited to  $\pm 10$  millivolts and the basic signal, which was of the order of 5 to 15 volts, required attenuation. Moreover, as the basic signal included a large DC component and interest was mainly with the velocity fluctuations which were expressed by the AC component of the signal, it was necessary to suppress the major part of the DC component prior to attenuation. This was accomplished by connecting in series a known DC voltage of opposite sign. The output of the recorder was to a magnetic tape on which the suppressed and attenuated bridge voltages were recorded in digital form. The data on the tape was registered in octal numbers ranging from 0000 to 2,047, corresponding to input voltages from -10 to +10 millivolts. A schematic flow diagram of a hypothetical signal is shown in Figure 43.



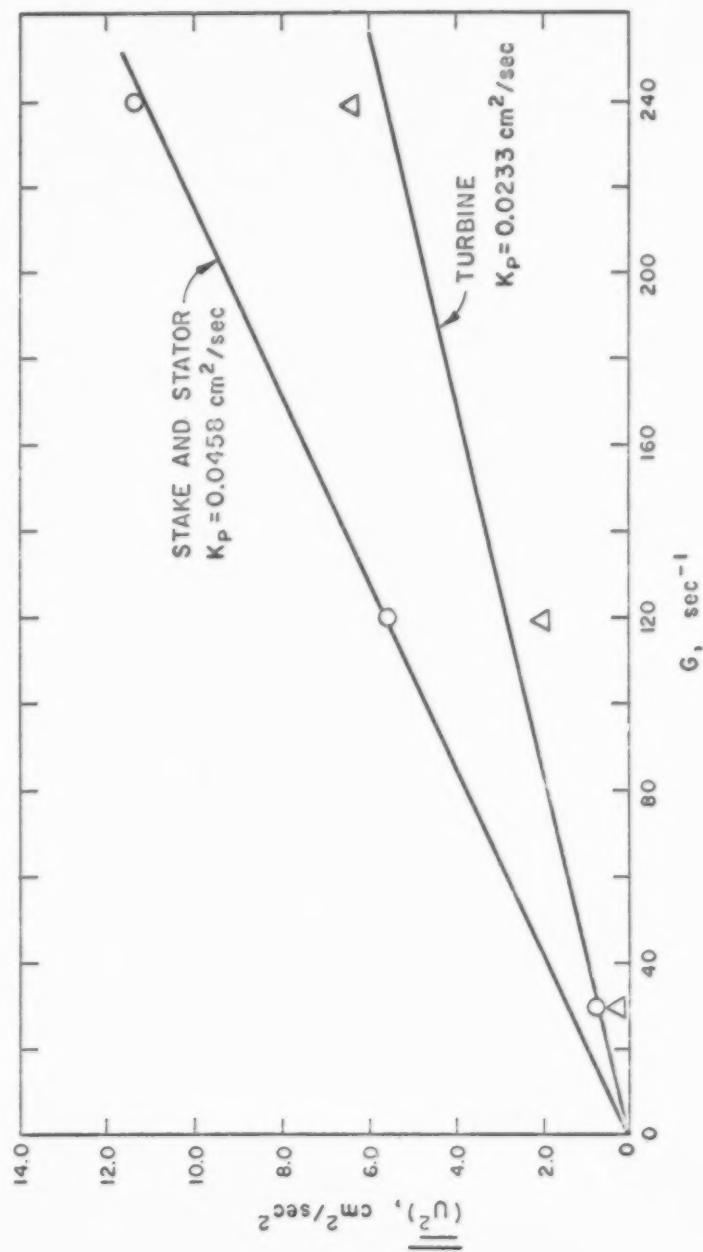


FIGURE 41. VARIATION OF THE VOLUME WEIGHTED AVERAGE  $\overline{U^2}$  WITH  $G$

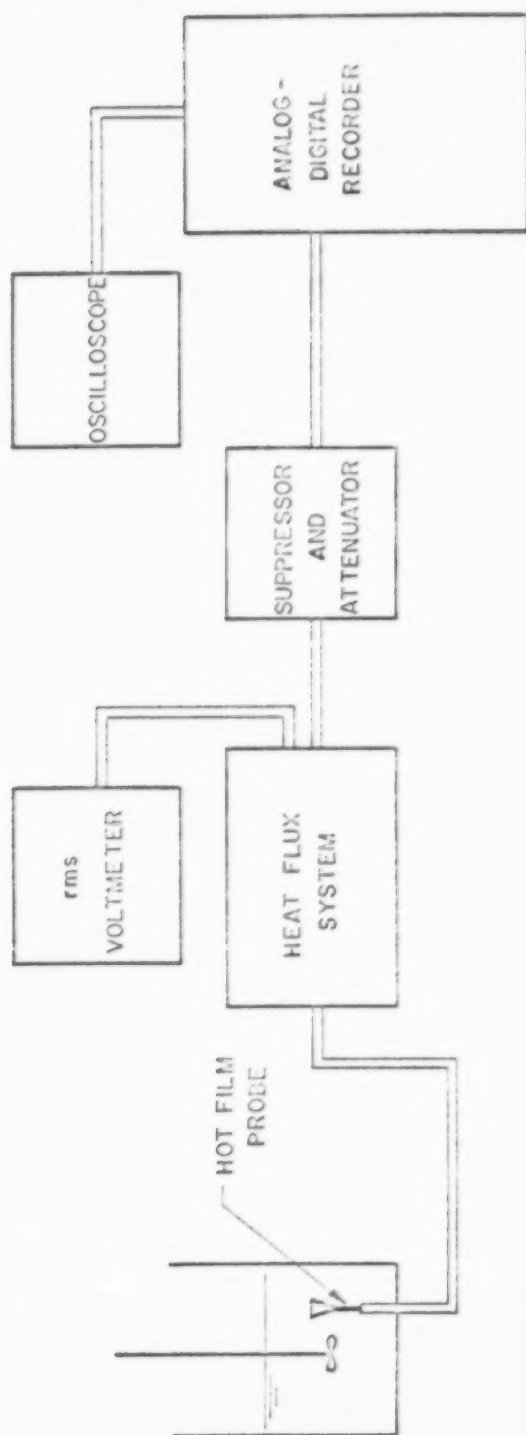


FIGURE 42. SCHEMATIC DIAGRAM OF TURBULENCE MEASURING APPARATUS

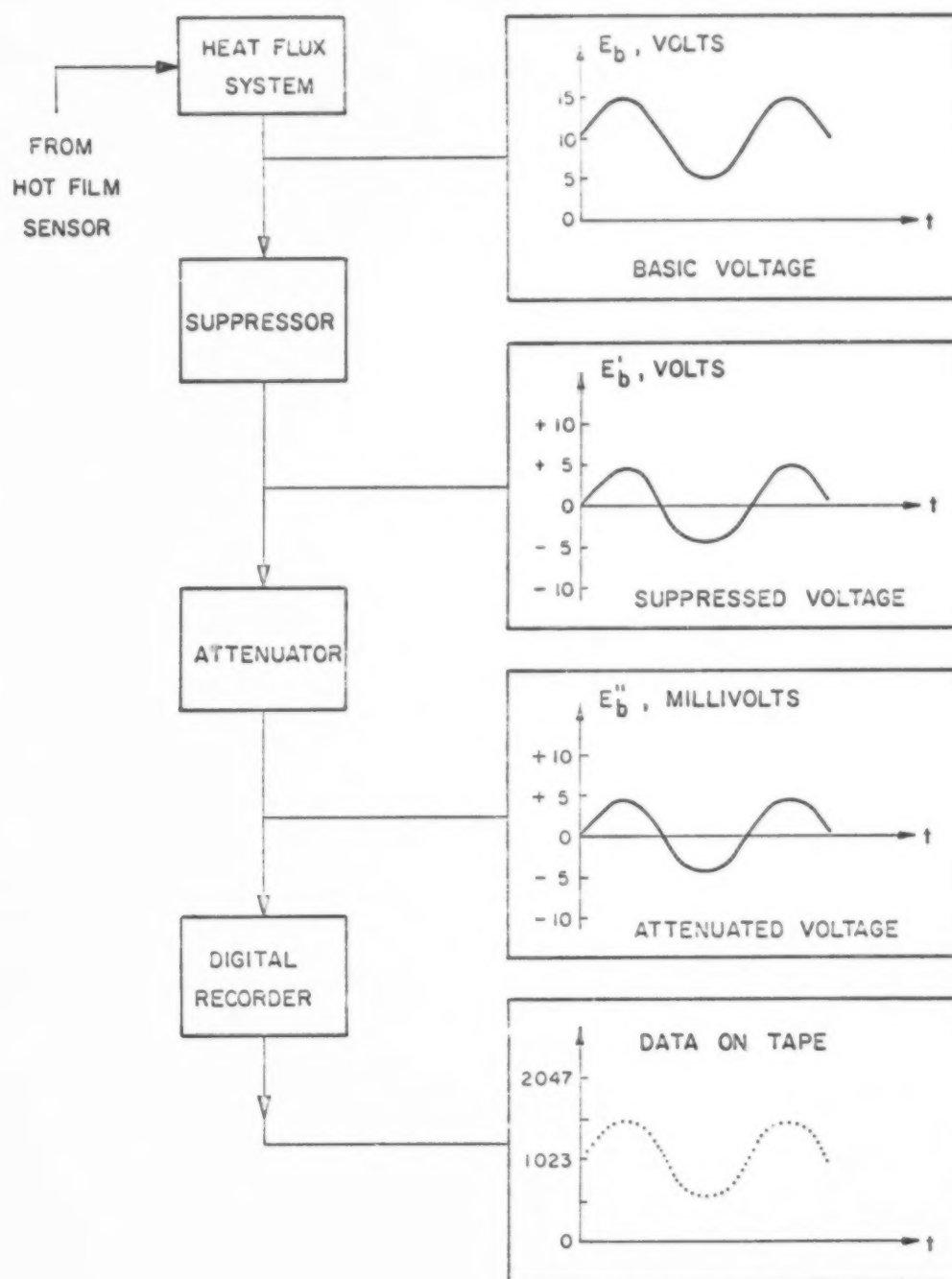


FIGURE 43. FLOW DIAGRAM OF SIGNAL FROM HEAT FLUX SYSTEM TO TAPE

### Computation Methods

The relation between the recorded data and the bridge voltage is given by

$$Z = 1.0235 \times 10^5 (E_D - V_S) \cdot F_A + 1.0235 \times 10^3 \quad (105)$$

where  $Z$  is the recorded data,  $E_D$  is the bridge voltage,  $V_S$  is the suppressing voltage, and  $F_A$  is the attenuation factor. The raw data tapes were first converted into "Direct Coupled System" blocked tapes that could be read by the IBM 7094. Because of limited memory capacity of the computer, each record was limited to 13,500 data points corresponding to a recording time of 7.2 or 28.8 sec depending on the sampling rate. To obtain the energy spectrum, a main computer program was written incorporating a set of subroutines for time series analysis [46]. These subroutines were modified to fit the specific needs of the current problem. A program listing is given in Appendix F. A block diagram of the program is shown in Figure 44.

The first subroutine called by the program is READ. It merely reads the data off the blocked tape. Each data point is a 12 octal digits word with three sample points packed per word. The next subroutine, UNPACK, unpacks each data word to three individual sample points. In the next step the velocity was calculated for each sample point. This was done by combining Equations 98 and 105

$$U_i = \left[ \frac{\left( \frac{Z_i}{1.0235 \times 10^5 - 10^{-2}} + V_S \right)^2}{F_A} - A \right] \cdot B \quad (106)$$

Subroutine DETRND was then called. This subroutine determines the mean value of the velocity, the slope of the straight-line of best fit (least square method), and calculates the deviation of each velocity from that straight-line. The method used for this calculation is as follows. First, the arithmetic mean velocity is calculated by

$$\bar{U} = \frac{\sum_{i=1}^N U_i}{N} \quad (107)$$

where  $N$  is the number of sample points.

The slope of the line of best fit is then calculated from

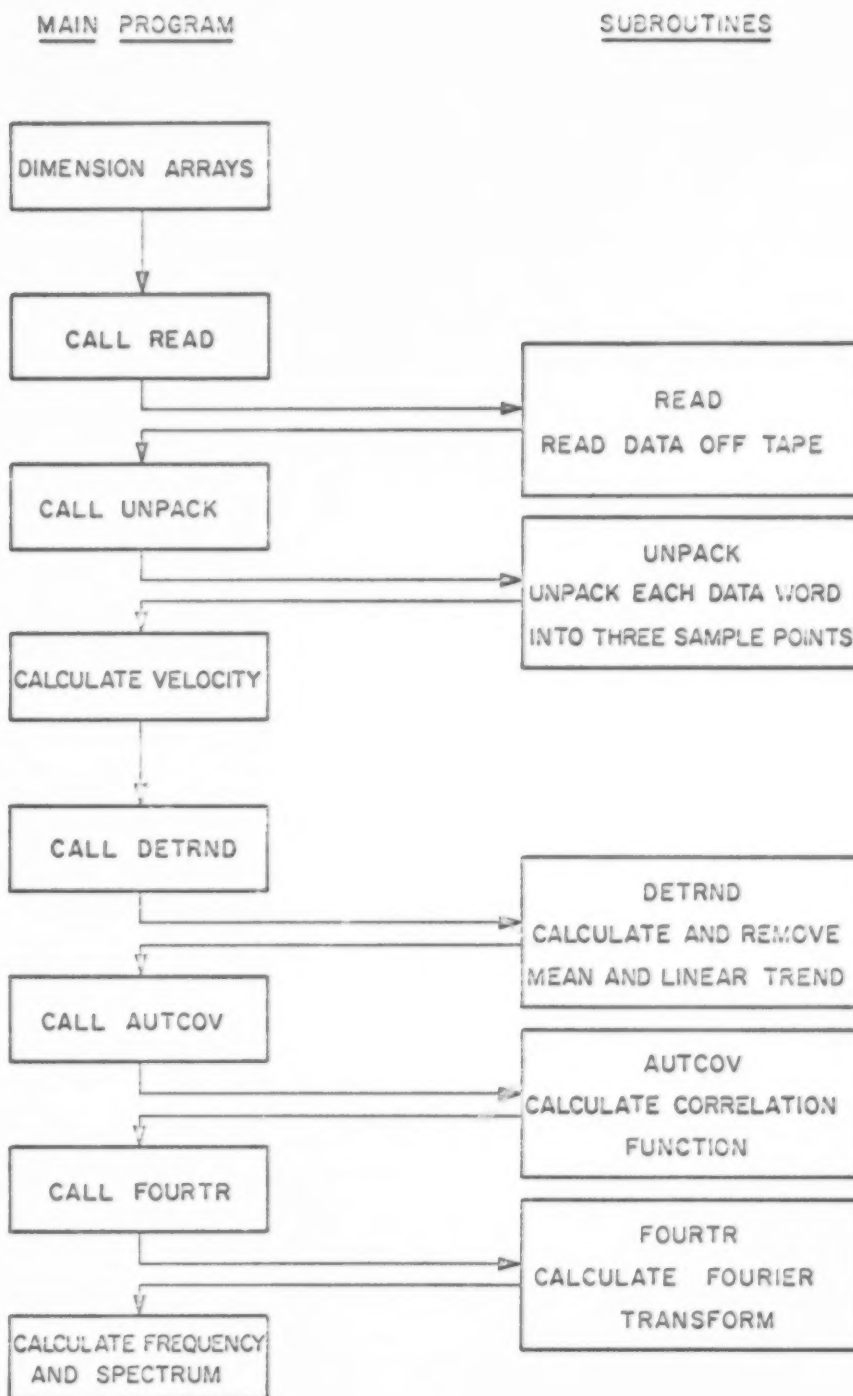


FIGURE 44. BLOCK DIAGRAM OF COMPUTER PROGRAM



$$S = -12 \frac{\sum_{i=1}^N (N - i + 1) U_i - \frac{1}{2} (N + 1) \sum_{i=1}^N U_i}{N (N^2 - 1)} \quad (108)$$

where  $S$  is the slope of the straight-line.

The deviations from the straight-line, i.e., the velocity fluctuations, are then calculated by

$$u_i = U_i - \bar{U} - S \left[ i - \frac{1}{2} (N + 1) \right] . \quad (109)$$

The reason for using this procedure for calculating the velocity fluctuations as opposed to a conventional deviation from the arithmetic mean is the following. During a single run which lasts only a few seconds, there may occur a steady change in mean velocity. If the velocity fluctuations were calculated by subtracting the arithmetic mean from the instantaneous velocity, the resulting velocity fluctuations would be very large, and not representative of the true fluctuations. The situation is illustrated in Figure 45 where  $u_a$ , the "conventional" velocity fluctuation is compared with  $u$ , the "true" velocity fluctuation used in the current analysis.

The correlation function for different time lags was then calculated using the AUTCOV subroutine. Time lags varied in steps of  $\rho\tau$ , where  $\rho$  is an integer, and  $\tau$  the sampling time interval. Thus for a time lag  $\Delta t = m\rho\tau$  the correlation function was calculated by

$$C(\Delta t) = \frac{1}{N - m} \sum_{i=1}^{N-m} u_i u_{i+m} - (\bar{u})^2 . \quad (110)$$

Finally the cosine Fourier transform of the correlation function was calculated by using the FOURTR subroutine. The computation method for the Fourier transform consisted of a numerical integration of the correlation function. The output of the subroutine was the unsmoothed normalized Fourier transform given by

$$F'(h) = \frac{1}{M} \left[ c(0) + 2 \sum_{q=1}^{M-1} C(q) \cos \frac{\pi q h}{M} + C(M) \cos \pi h \right] \quad (111)$$

$$h = 0, 1, \dots, M$$

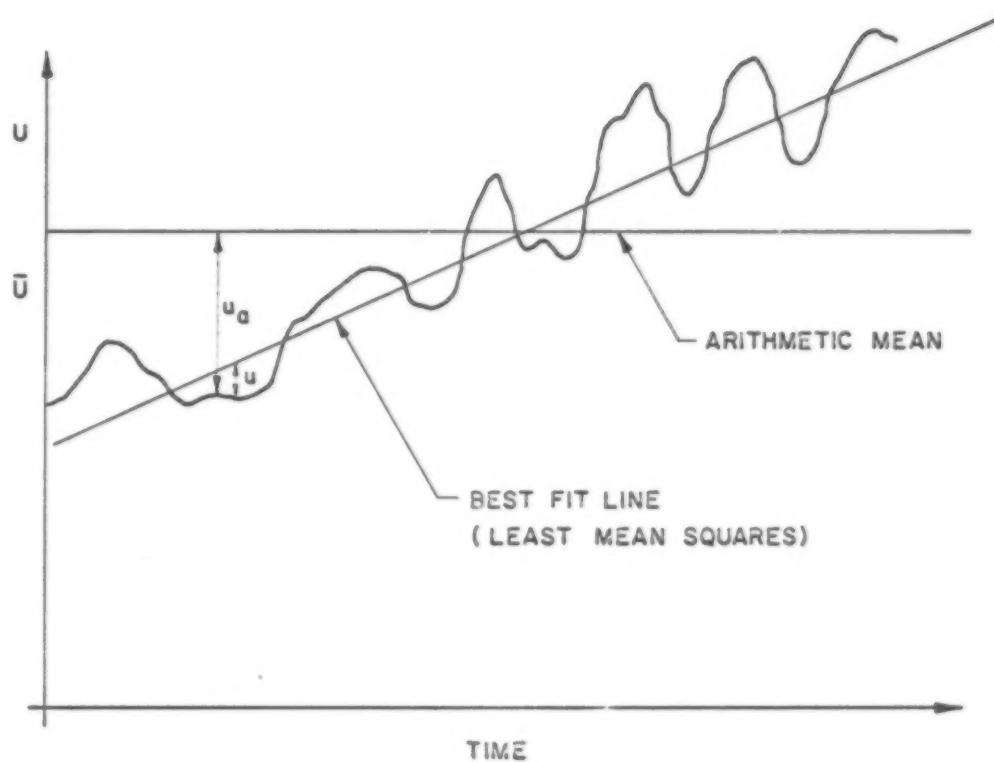


FIGURE 45. "TRUE" AND "APPARENT" VELOCITY FLUCTUATIONS

where  $M$  is the number of points in the correlation array or the number of time lags used in calculating the correlation function.

The frequency,  $f$ , in cps is related to  $h$  by

$$f = \frac{h}{2 M \tau} \quad (112)$$

The Fourier transform was then smoothed by the Hanning [44] process using factors of 0.25, 0.5, and 0.25. The smoothed energy spectrum function was calculated by

$$E(f) = 2 M \tau F(h) \quad (113)$$

where  $F(h)$  is the smoothed Fourier transform. Some additional comments regarding the spectrum measurements and calculations are given in Appendix G.

To obtain a complete and accurate picture of the turbulence field in the investigated reactor, a series of 60 runs was planned. Unfortunately, the digital recorder was not functioning properly and most of the tapes produced had permanent read errors. These tapes could not be blocked and processed. Since a relatively long computer time was required before the read error could be detected, it was impossible to obtain all the results as planned. From more than 150 runs only 17 records could be blocked and processed and these results are presented herein. Although there are not sufficient data for a complete analysis, some significant conclusions may be drawn.

#### Results of Turbulence Measurements

The correlation functions calculated by Equation 110 are shown in Figures 46 through 50. The first two figures illustrate the dependence of the correlation function on the total power input (expressed by the rms velocity gradient). They also indicate that the mean square velocity fluctuations increase linearly with  $G$  as was assumed in Equation 84. The rate at which the correlation function approaches zero is greater at higher values of  $G$ .

In Figures 48 through 50, the correlation functions for the two paddle types are compared. The stake and stator paddle seems to produce larger values of  $u^2$  with a greater rate of change in the correlation function.

Turbulence energy spectra are shown in Figures 51 through 54. The first figure shows the effect of total power input on the spectrum. The total areas under the spectral curves which equal  $u^2$ , appear to increase with  $G$ . In addition, it seems that the spectra extend to higher frequencies as  $G$  increases, indicating that larger amounts of

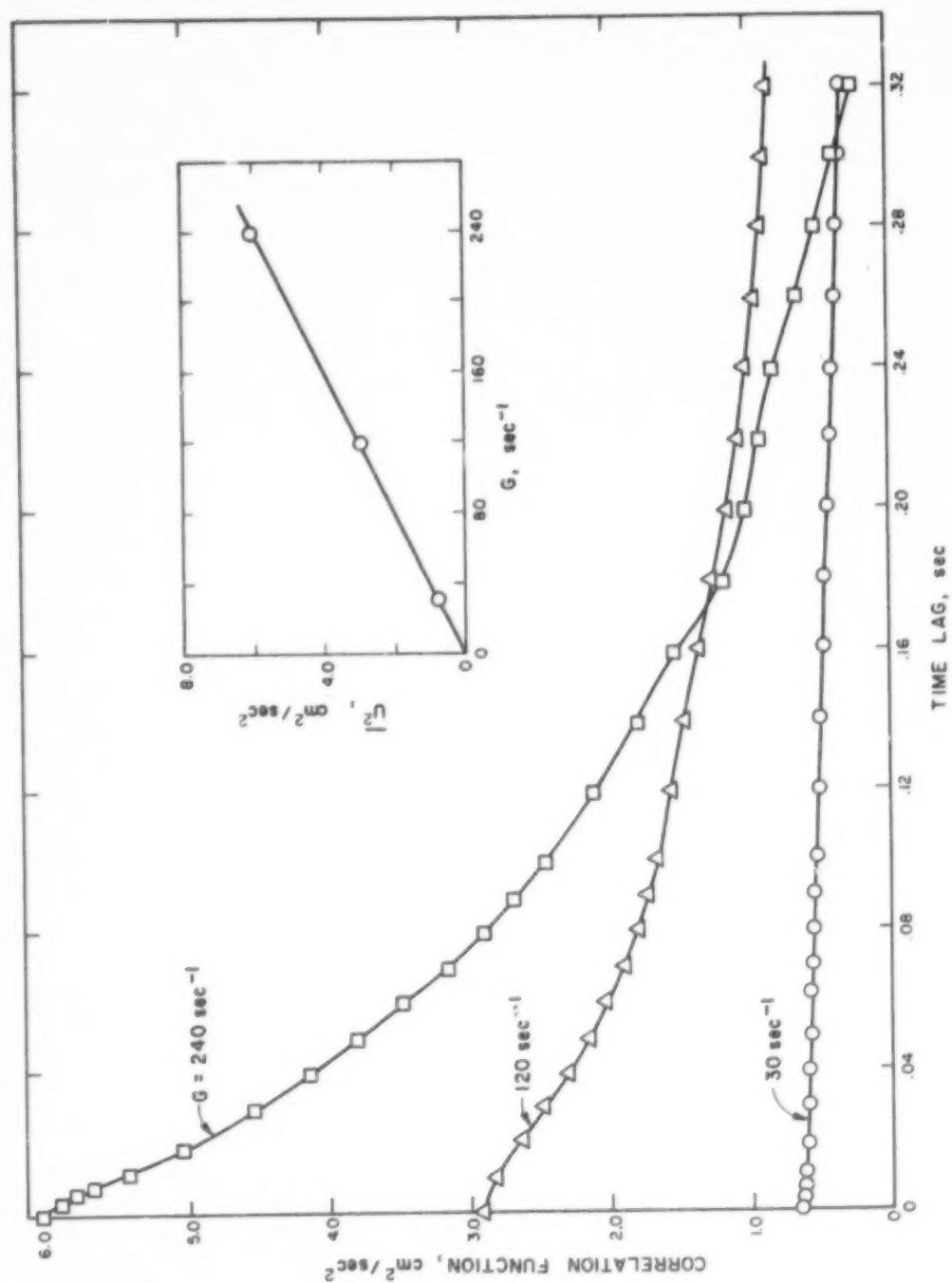


FIGURE 46. CORRELATION FUNCTION AT 5-1/4 in. FROM CENTER; 3 in. BELOW MID-DEPTH; TURBINE PADDLE

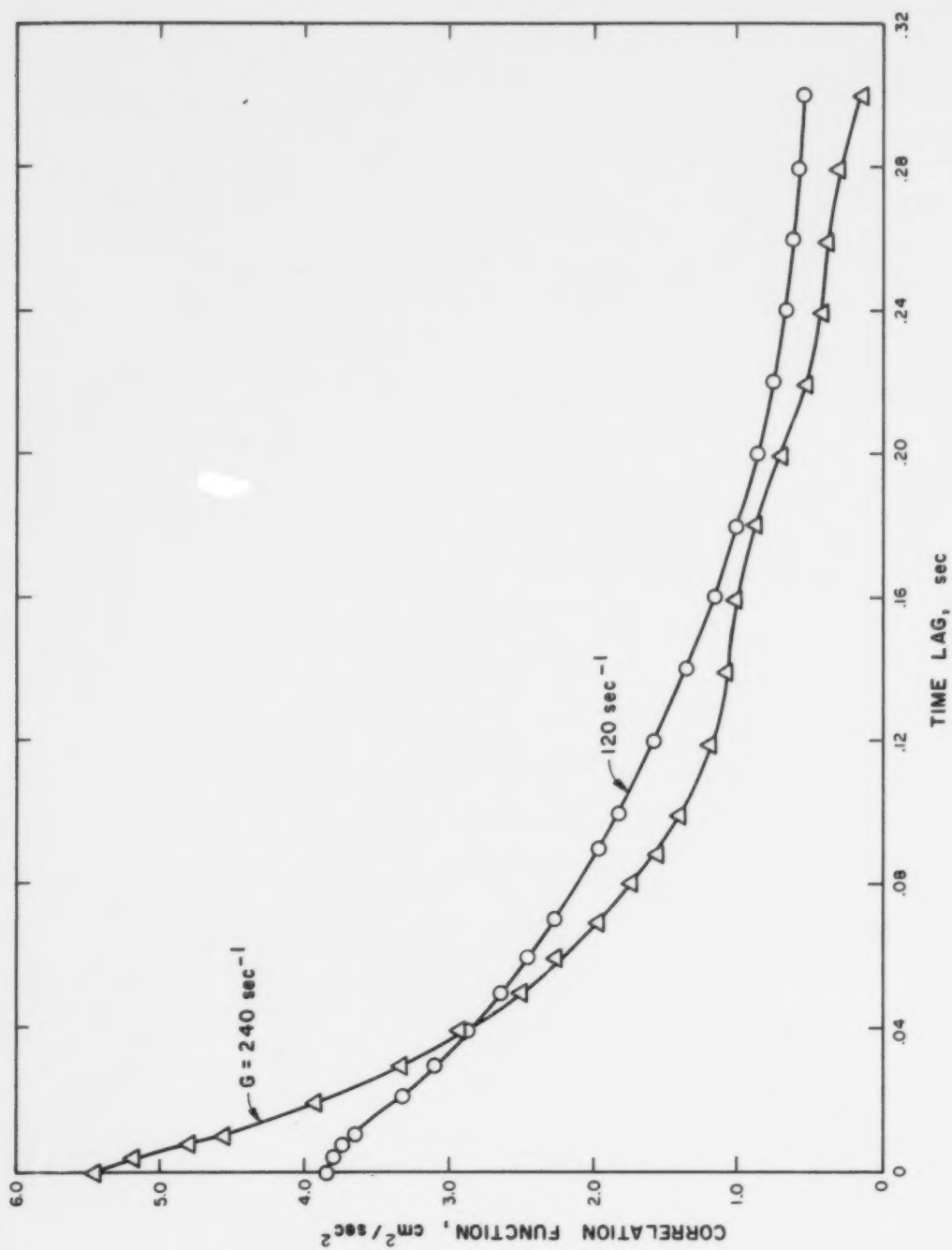


FIGURE 47. CORRELATION FUNCTION AT 3-1/4 in. FROM CENTER, 3 in. BELOW MID-DEPTH, TURBINE PADDLE

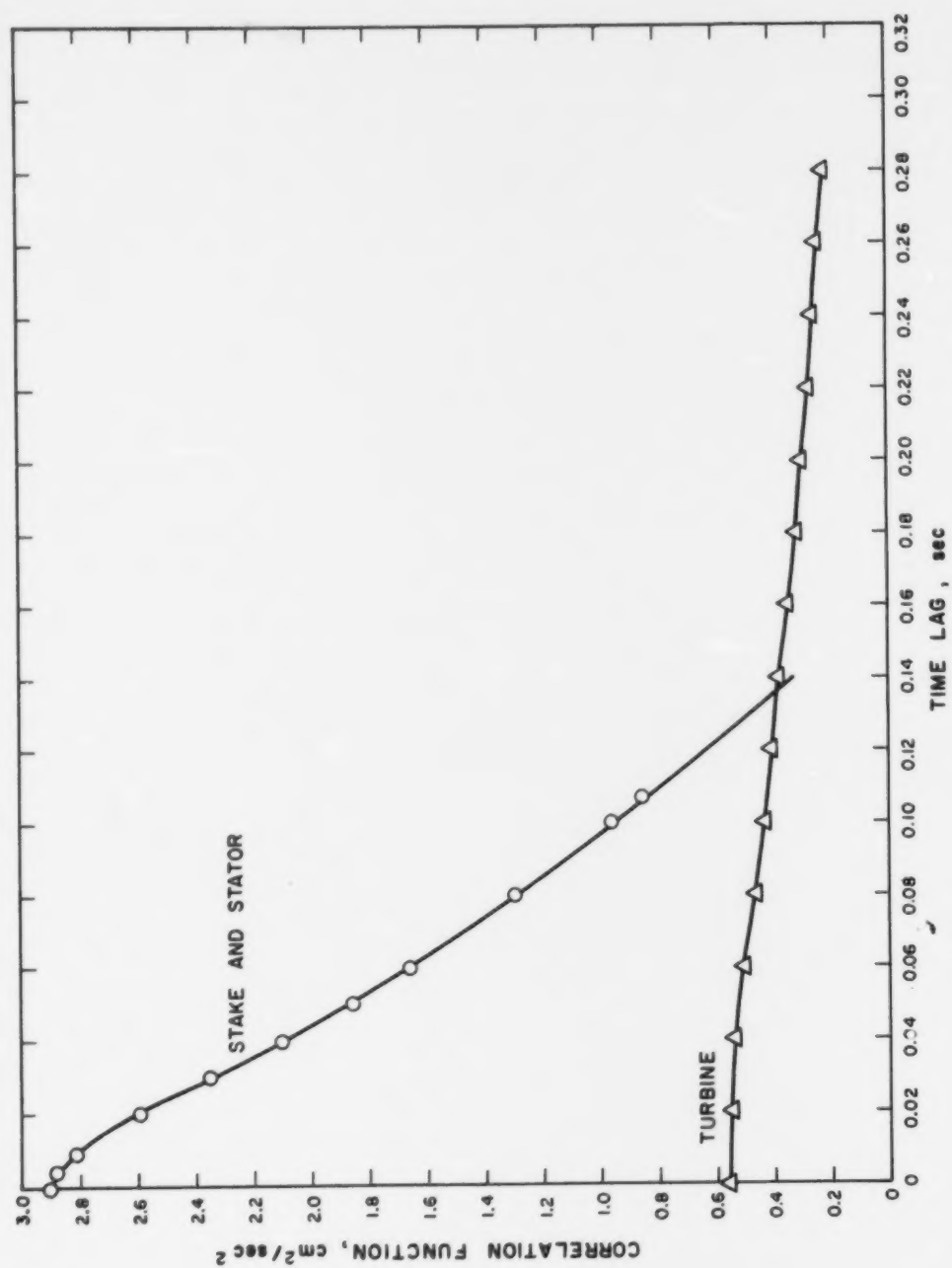


FIGURE 48. CORRELATION FUNCTION AT 3-1/4 in. FROM CENTER, MID-DEPTH  
 $G = 30 \text{ sec}^{-1}$



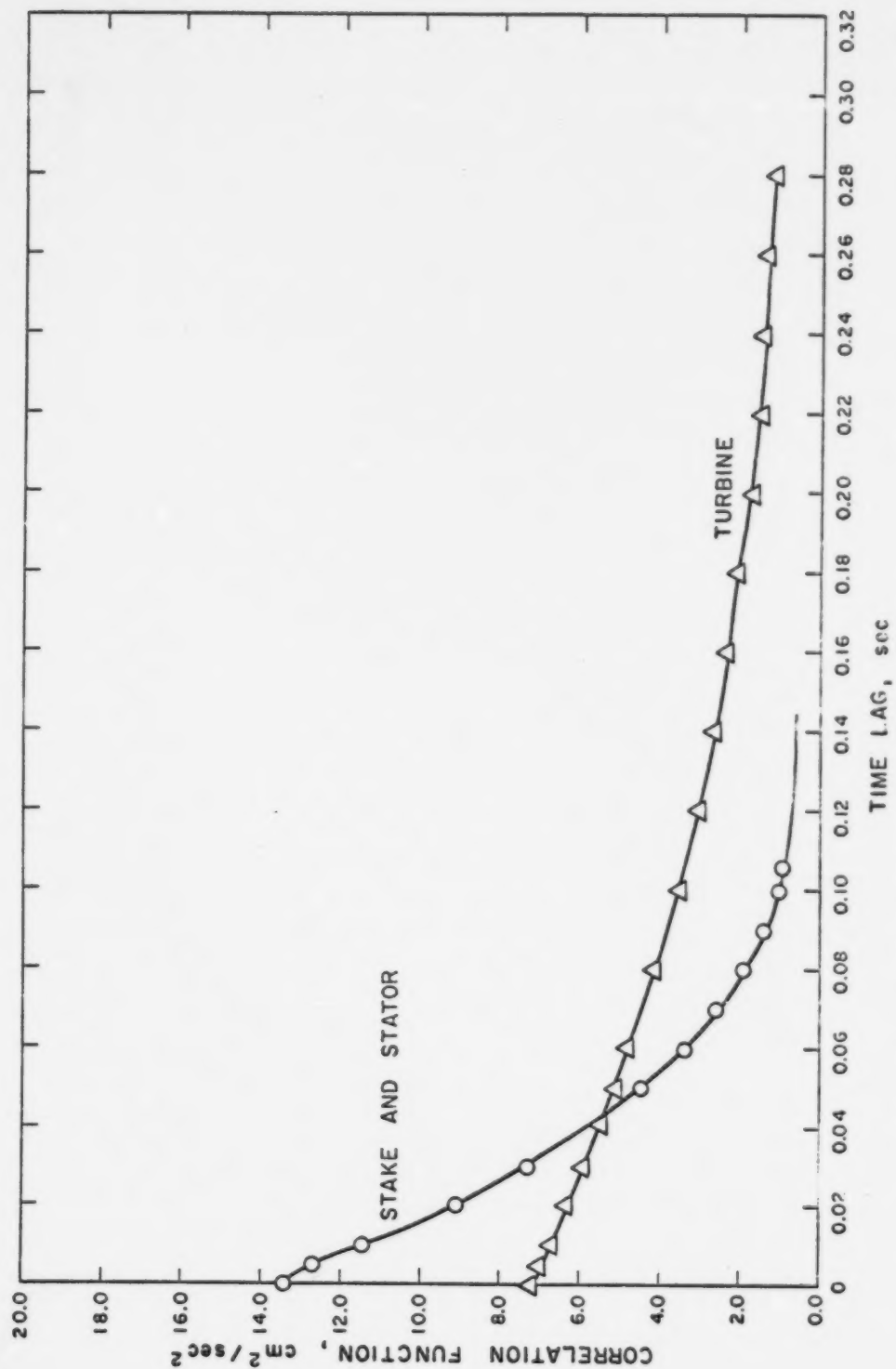


FIGURE 49. CORRELATION FUNCTION AT 3-1/4 in. FROM CENTER, MID-DEPTH.  $G=120\text{sec}^{-1}$

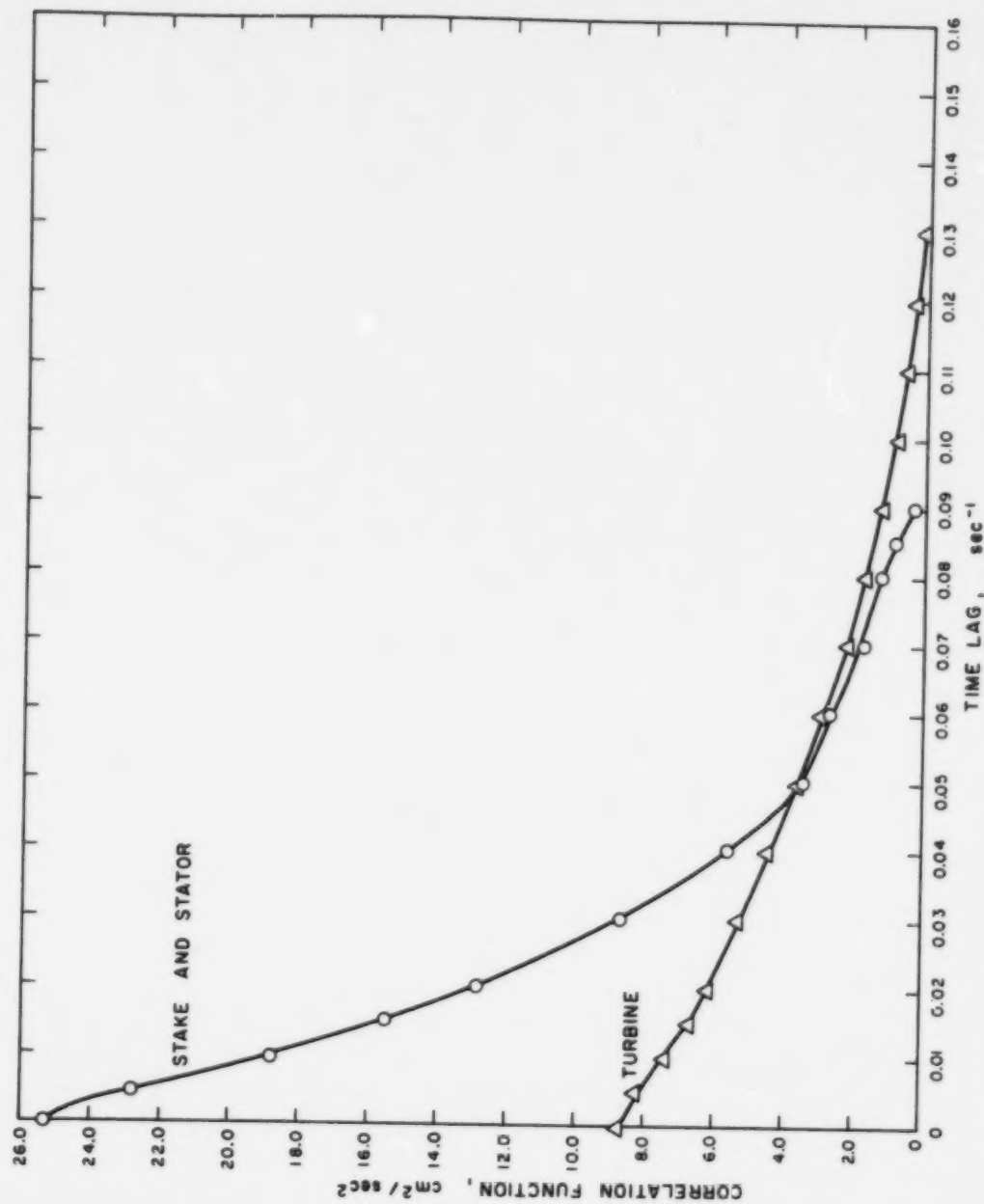
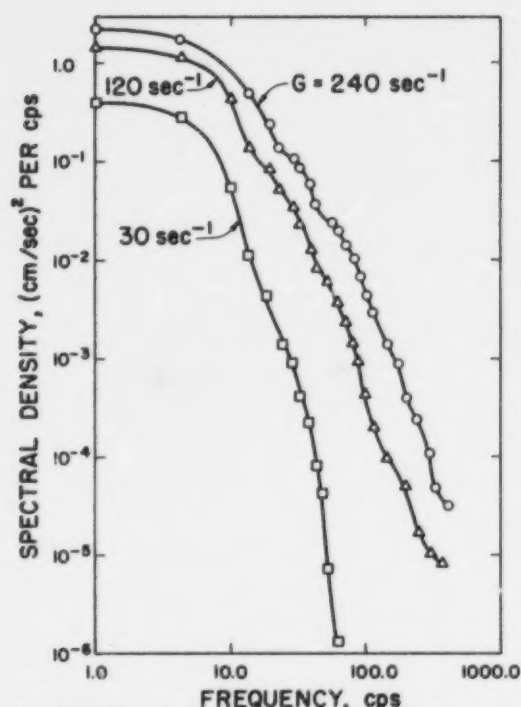
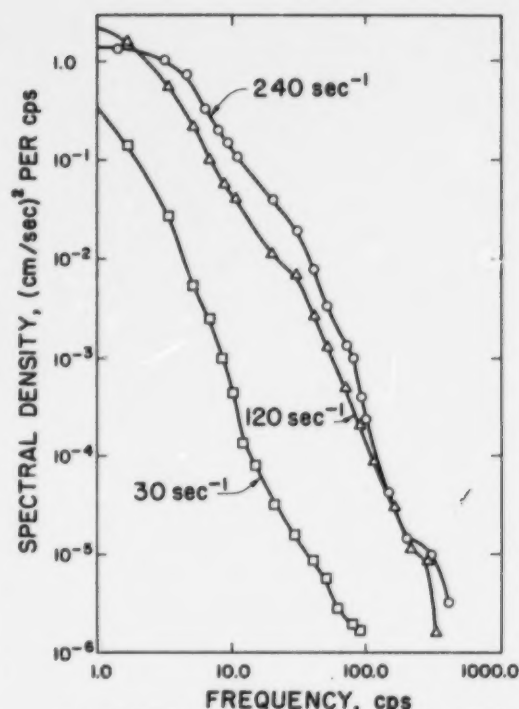


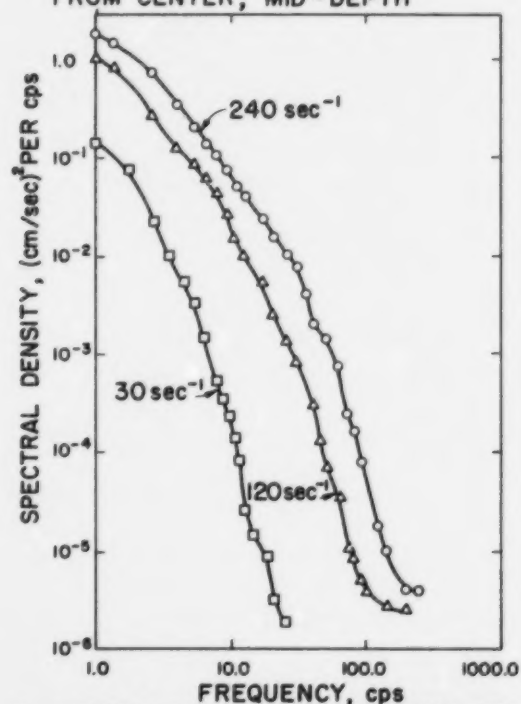
FIGURE 50. CORRELATION FUNCTION AT 3-1/4 in. FROM CENTER, MID-DEPTH.  $G = 240 \text{ sec}^{-1}$



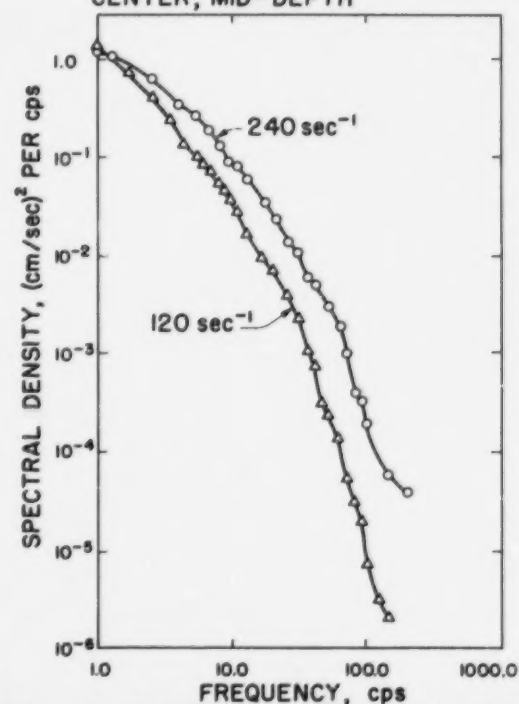
A. STAKE AND STATOR PADDLE; 3-1/4 in. FROM CENTER, MID-DEPTH



B. TURBINE PADDLE; 3-1/4 in. FROM CENTER, MID-DEPTH



C. TURBINE PADDLE; 5-1/4 in. FROM CENTER, 3 in. BELOW MID-DEPTH



D. TURBINE PADDLE; 3-1/4 in. FROM CENTER, 3 in. BELOW MID-DEPTH

FIGURE 5I. VARIATION OF ENERGY SPECTRUM WITH POWER INPUT

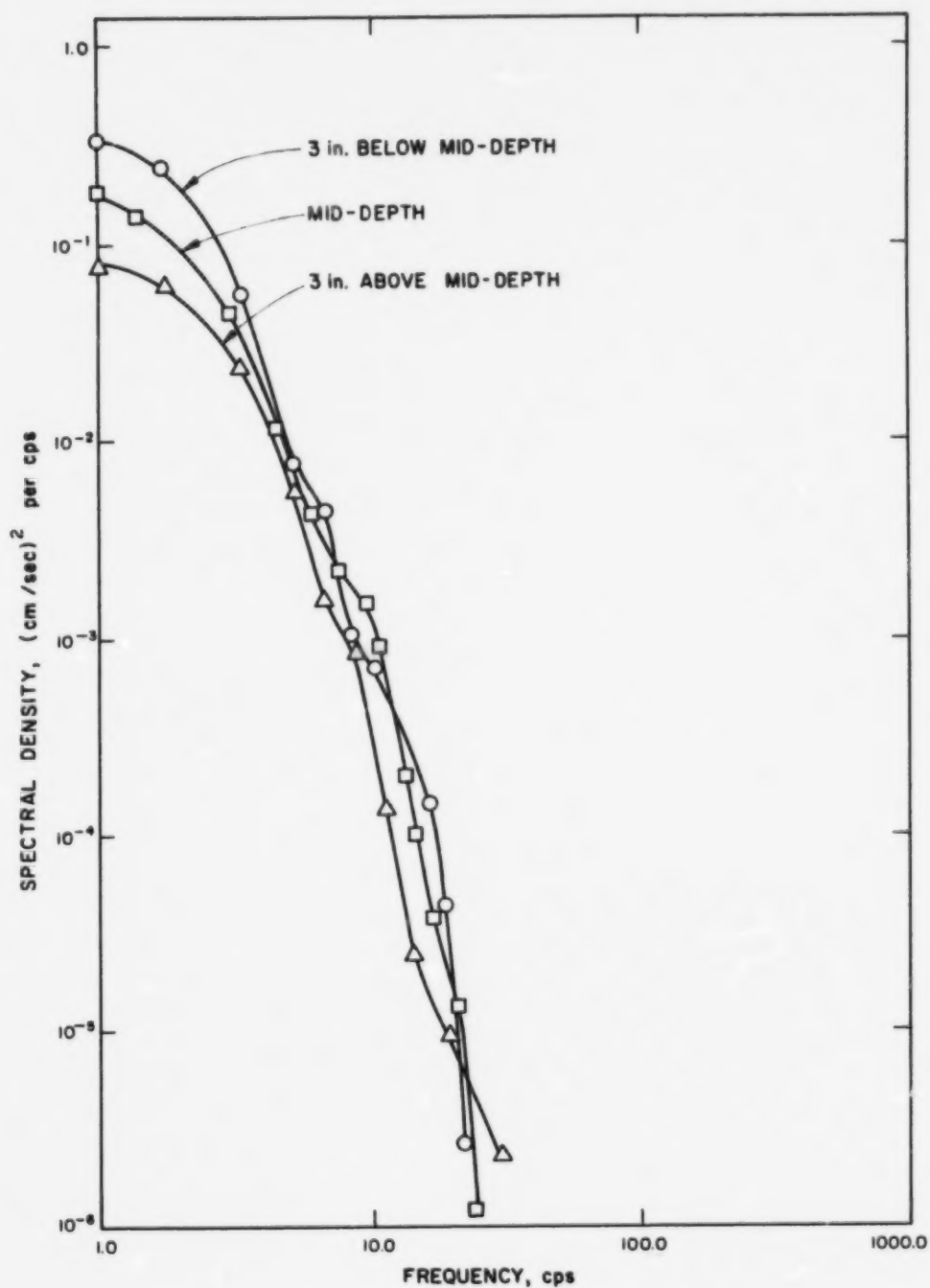


FIGURE 52. VARIATION OF ENERGY SPECTRUM WITH DEPTH  
(STAKE AND STATOR,  $G = 30 \text{ sec}^{-1}$ ,  $5 - 1/4$  in. FROM CENTER)

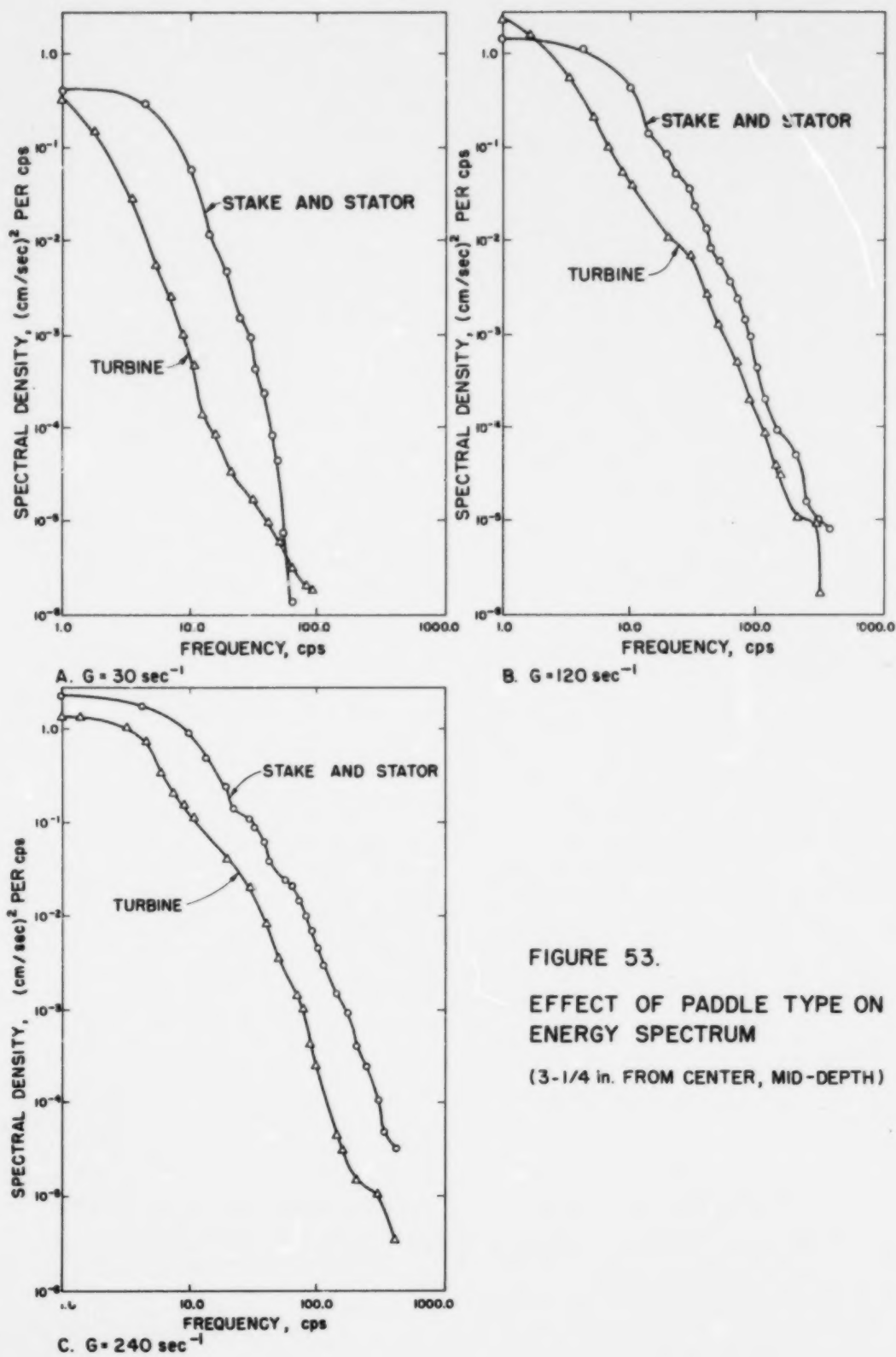


FIGURE 53.

EFFECT OF PADDLE TYPE ON  
ENERGY SPECTRUM

(3-1/4 in. FROM CENTER, MID-DEPTH)

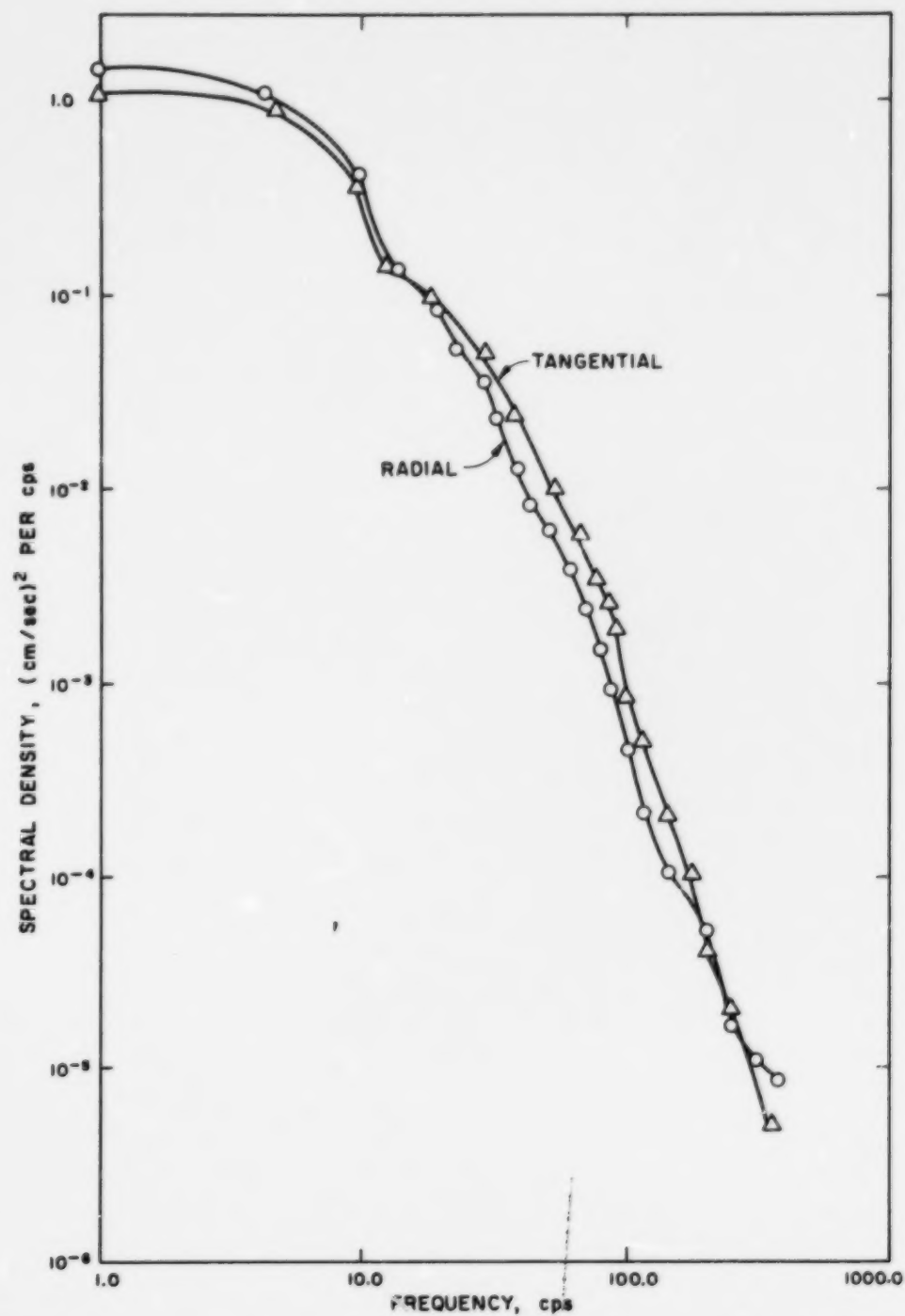


FIGURE 54. VARIATION OF ENERGY SPECTRUM WITH SENSOR ORIENTATION  
( $G = 120 \text{ sec}^{-1}$ , STAKE AND STATOR, 3-1/4 in. FROM CENTER, MID-DEPTH)



energy are carried by small eddies as the total power input increases. These results are in accordance with the theory which predicts the following relationship between the power input and the Kolmogoroff microscale of turbulence [13]

$$\lambda_0 = \left( \frac{\nu^3}{\epsilon} \right)^{\frac{1}{4}} \quad (114)$$

where  $\lambda_0$  is the microscale.

The variations of the energy spectra with depth at a given power input is shown in Figure 52. Although the value of  $u^2$  seems to increase with elevation, the variations are minor and the shapes of the spectra are essentially the same at all vertical locations.

Figure 53 is a comparison of the energy spectra generated by the different paddles at equal power inputs. In all cases the stake and stator paddle gave larger values of  $u^2$  as indicated by the larger areas under the spectrum curves. The difference in the spectra tends to diminish at higher frequencies. According to the theory, the rate of energy dissipation depends on the high frequency portion of the spectrum. As in each of these plots the power dissipation is the same for the two paddles (same G), the high frequency portions of the spectra were expected to be the same.

Throughout the theoretical parts of this investigation, it was assumed that the turbulence was isotropic. Some indication to the validity of this assumption is provided in Figure 54. The two energy spectra shown were measured at a single location with equal power input, the only difference being the orientation of the hot-film anemometer sensor. A 90° rotation of the sensor showed very little change in spectrum indicating that the turbulence field under the tested conditions was isotropic. This result is in accordance with the Kolmogoroff theory of locally isotropic flow, which suggests that at high Reynolds numbers the small scale turbulent motions are isotropic. According to Shinnar [47], this condition applies to stirred tanks if the impeller Reynolds number is greater than about  $10^4$ . The impeller Reynolds number is defined by

$$R_e = \frac{D^2 N}{\nu} \quad (115)$$

where  $D$  is the impeller diameter in cm,  $N$  is the impeller speed in revolutions per minute, and  $\nu$  the kinematic viscosity of the fluid. With  $D = 10$  cm,  $N = 40$  rpm, and  $\nu = 10^{-2}$  cm<sup>2</sup>/sec the Reynolds number is  $40 \times 10^5$ , and the Kolmogoroff theory applies.

## VI. ANALYSIS OF RESULTS

### GENERAL VALIDITY OF PROPOSED THEORY

The theoretical treatment of flocculation kinetics presented in Chapter III resulted in a working equation which, for a single compartment completely mixed reactor, is

$$\frac{n_1^0}{n_1^1} = \frac{1 + K_F K_S K_P n_1^0 T G}{1 + K_B T G^2} \quad (85)$$

The flocculation constant,  $K_P$ , given by

$$K_P = 4\pi\alpha K_1 R_1^3 \quad (86)$$

depends primarily on the chemical conditions prevailing during the process. It is directly proportional to the aggregation collision ratio ( $\alpha$ ), and inversely proportional to the floc density. The energy spectrum coefficient,  $K_S$ , represents the effect of the shape of the energy spectrum on the diffusion coefficient. It depends on the shape of the spectrum at the frequencies most pertinent in the flocculation process, i.e., frequencies which correspond to eddies of the magnitude of the collision radius. If it is assumed that the shape of the spectrum over the relevant frequency range is independent of the power input, then for a given stirring device  $K_S$  is a constant. The paddle performance coefficient,  $K_P$ , is the ratio between the volume weighted average value of  $u^2$  and  $G$ . Experimental results shown in Figures 31-34, and 40 provide ample evidence that for a given type of stirring device,  $K_P$  can be considered as a constant. The breakup coefficient,  $K_B$ , which was defined by

$$K_B = \frac{BK_P^2 R_1^3 K_1}{K_2} \quad (87)$$

is a measure of floc strength (or weakness) and of the tendency of the stirring device to cause floc breakup. For a given stirring device and fixed chemical conditions  $K_B$  is a constant.

Under these conditions, and at constant  $T$ , Equation 85 can be reduced to

$$\frac{n_1^0}{n_1^1} = \frac{1 + a G}{1 + b G^2} \quad (116)$$

where

$$a = K_F K_S K_P n_1^0 T \quad (117)$$

and

$$b = K_B T \quad (118)$$

as the experimental data were presented as  $n_1^0/n_1^1$  versus  $G$  for constant  $T$  and  $n_1^0$  (cf., Figures 11 through 14), the constants  $a$  and  $b$  of Equation 116 were estimated by fitting curves to these data points. The theoretical curves, their equations, and the experimental data are shown in Figures 55 and 56. The values of the empirical constants  $a$  and  $b$  are given in Table III. These values are also plotted against  $T$  in Figure 57. According to Equations 117 and 118, both  $a$  and  $b$  should be linear functions of  $T$ . As can be seen from Figure 57 a linear relationship between  $b$  and  $T$  does exist, except for a single deviation at  $T = 12$  min, with the stake and stator paddle. The deviation is probably due to experimental error. The deviations from linear relationships are more prominent in the  $a$  versus  $T$  plot. The experimental data suggest that  $a/T$ , which according to Equation 117 should be a constant, varies with  $T$ , indicating that the product  $K_F K_S K_P n_1^0$  decreases with increasing residence time. Since  $K_S$ ,  $K_P$ , and  $n_1^0$  are not time dependent, the conclusion is that  $K_F$ , the flocculation constant, has some dependence on  $T$ . This can be explained by considering the variations in the aggregation-collision ratio,  $\alpha$ , with time. This ratio is certainly dependent on the chemical properties of the system and the geometry of the flocs. As  $T$  increases the flocs tend to assume a more regular shape, they are more compact, and the probability of a successful collision decreases. Thus the decrease in  $\alpha$  with  $T$  results in a reduced  $K_F$ .

The relatively good fitness of the experimental data to the theoretical curves and the foregoing analysis showing the independence of  $K_B$ ,  $K_S$ , and  $K_P$  on either  $T$  or  $G$ , indicates the general validity of Equation 85 and the theoretical arguments upon which it is based.

#### TURBULENCE AND FLOCCULATION KINETICS

The theoretical analysis presented in Chapter III showed that the effect of the turbulence on flocculation kinetics is expressed by the effective diffusion coefficient,  $D_{eff}$ . It was also shown in Figure 5 that  $D_{eff}$  was a function of the product of the entrainment

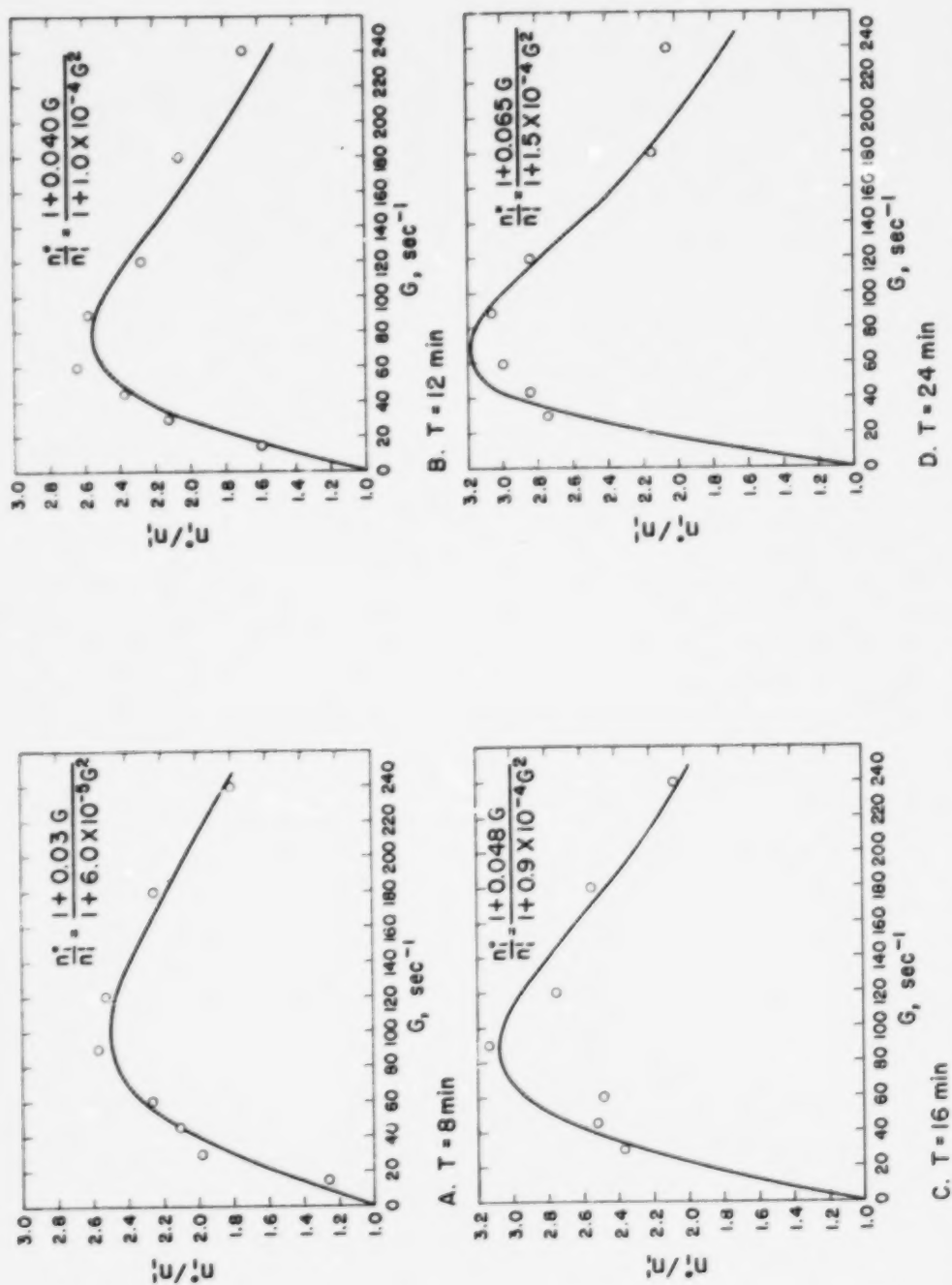


FIGURE 55. CONFORMANCE OF EXPERIMENTAL DATA TO THEORY, STAKE AND STATOR PADDLE

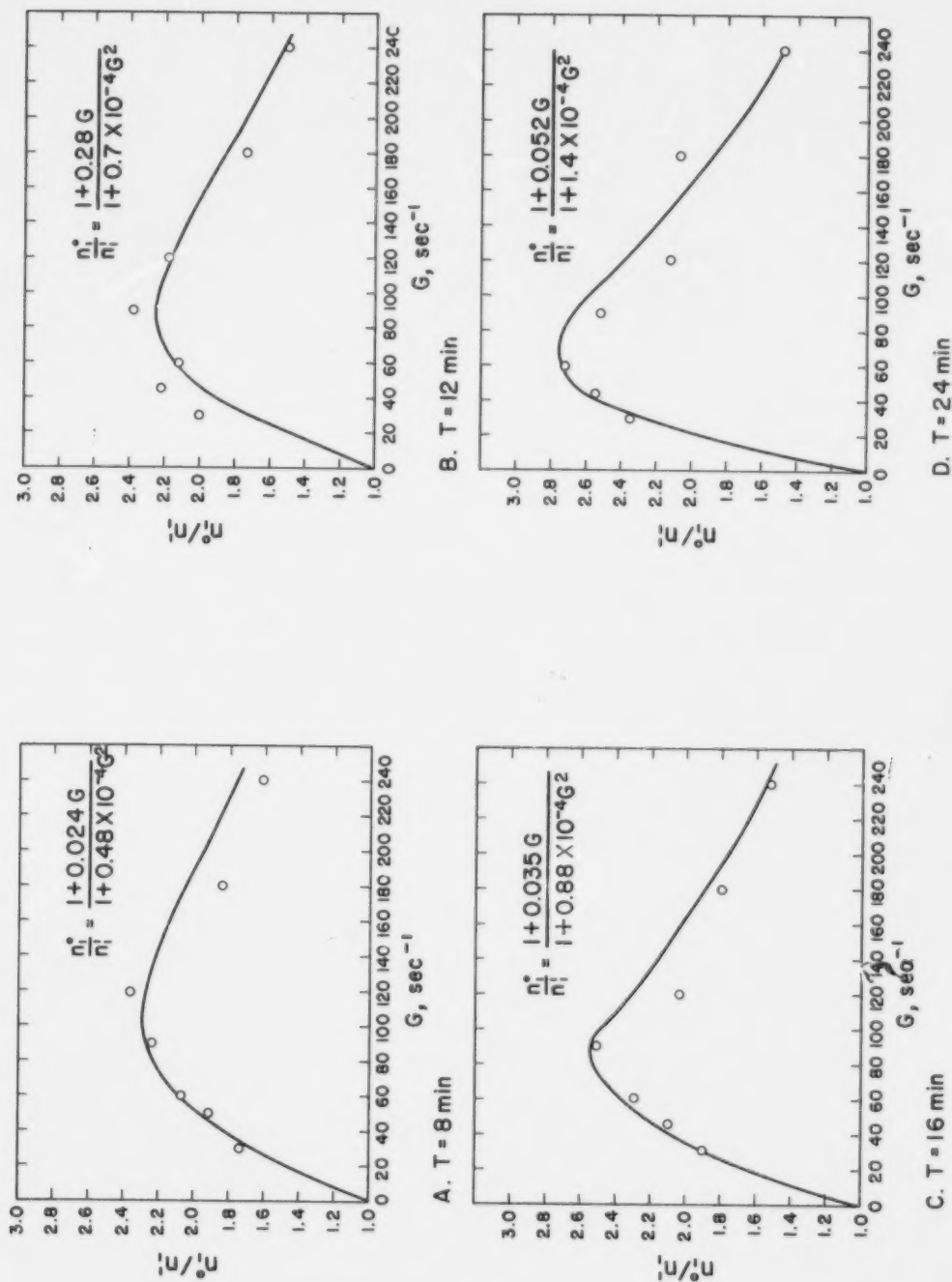


FIGURE 56. CONFORMANCE OF EXPERIMENTAL DATA TO THEORY. TURBINE PADDLE



TABLE III  
EMPIRICAL TIME DEPENDENT PARAMETERS OF  
THE FLOCCULATION EQUATION

| No. | Residence<br>Time<br>min | Paddle Type      | a<br>sec | $10^4$ b<br>sec <sup>2</sup> |
|-----|--------------------------|------------------|----------|------------------------------|
| 1   | 8                        | Stake and Stator | 0.030    | 0.60                         |
| 2   | 12                       | Stake and Stator | 0.040    | 1.00                         |
| 3   | 16                       | Stake and Stator | 0.048    | 0.90                         |
| 4   | 24                       | Stake and Stator | 0.065    | 1.50                         |
| 5   | 8                        | Turbine          | 0.024    | 0.48                         |
| 6   | 12                       | Turbine          | 0.028    | 0.70                         |
| 7   | 16                       | Turbine          | 0.035    | 0.88                         |
| 8   | 24                       | Turbine          | 0.052    | 1.40                         |



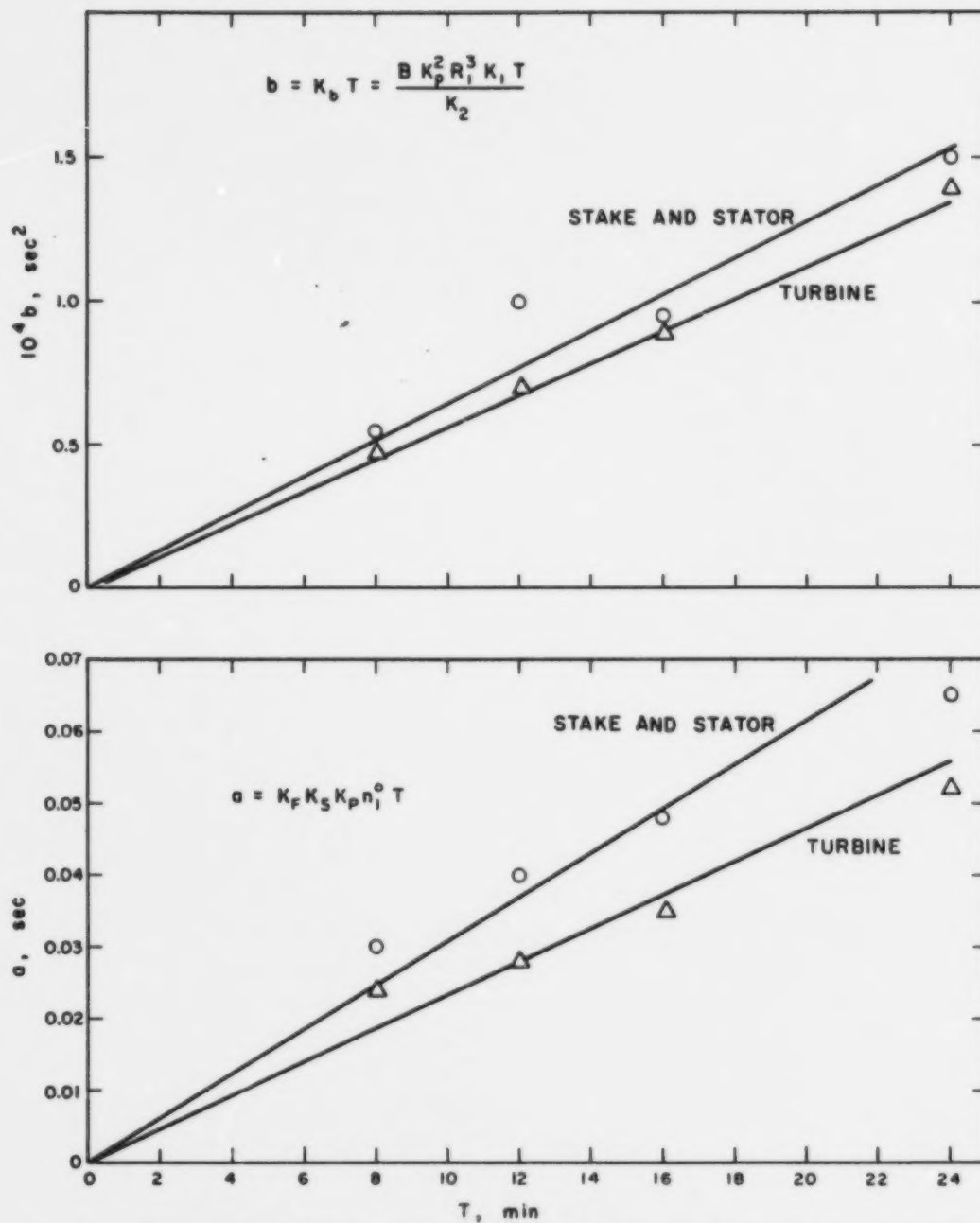


FIGURE 57. EMPIRICAL CONSTANTS OF THE FLOCCULATION EQUATION

factor and the spectral density  $[R(f)]$  in the vicinity of  $f_0$ . To support further the validity of the proposed model, an attempt was made to compare the values of  $D_{eff}$  obtained from direct spectrum measurements with those deduced from the flocculation equations. According to Equations 78 and 84 the effective diffusion coefficient is related to the flocculation equation constants by

$$D_{eff} = K_s K_p R_F^2 G \quad (119)$$

Hence the experimental value of  $D_{eff}$  can be estimated if the product  $K_s K_p$  is known.

From Figure 57 the product  $K_s K_p K_F n_1^0$  was found to be  $5.14 \times 10^{-5}$  and  $3.86 \times 10^{-5}$  for the stake and stator and the turbine paddles respectively. The product  $K_s K_p$  can be calculated using estimated values for  $n_1^0$  and  $K_F$ . The number concentration of primary particles in the raw water is estimated from their weight concentration of 27.0 mg/l, and effective diameter of 3.0 microns and a specific gravity of 2.6. Thus  $n_1^0 \approx 0.75 \times 10^6$  particles per  $cm^3$ .  $K_F$  is obtained from Equation 86, taking  $R_1 = 1.5$  microns, and  $\alpha K_1 \approx 1$ . Thus  $K_F \approx 1 \times 10^{-11} cm^3$ . It should be noted that the exact values of  $n_1^0$  and  $K_F$  are not of major concern as only relative values of  $D_{eff}$  are considered. Hence the empirical values of  $K_s K_p$  are 6.06 and 5.18 for the stake and stator and turbine paddles respectively. With  $R_F$  values from Figure 28 and  $R_1$  of 1.5 microns, values of  $D_{eff}$  were calculated and are shown in Table IV. The decrease of  $D_{eff}$  with  $G$  is because the floc size, which is inversely proportional to  $G$ , appears in the second power in Equation 119.

TABLE IV  
DIFFUSION COEFFICIENTS CALCULATED FROM  
FLOCCULATION DATA

| G<br>sec <sup>-1</sup> | R <sub>F</sub><br>microns | D <sub>eff</sub> , cm <sup>2</sup> /sec |                       |
|------------------------|---------------------------|-----------------------------------------|-----------------------|
|                        |                           | Stake and Stator                        | Turbine               |
| 30                     | 170                       | $1.33 \times 10^{-2}$                   | $1.01 \times 10^{-2}$ |
| 120                    | 65                        | $7.72 \times 10^{-3}$                   | $5.86 \times 10^{-3}$ |
| 240                    | 25                        | $2.28 \times 10^{-3}$                   | $1.73 \times 10^{-3}$ |

In calculating the effective diffusion coefficients from the measured energy spectrums two difficulties are encountered. First, not enough data are available for calculating a diffusion coefficient which is representative of the entire reactor volume. This constitutes

a rather severe limitation in view of the nonhomogeneous characteristics of the turbulence field. Second, the diffusivity filter function,  $\sin 2\pi ft/2\pi f$  is unknown unless the diffusion time,  $t$ , is assumed. To overcome these difficulties, the energy spectra measured at the center point of the reactor were selected as typical spectra for the entire volume. The diffusion time is estimated by considering the time it takes for a primary particle to move from the dispersed phase to the surface of a floc. The distance of concern is of the same magnitude as the floc size, and the velocity is that of an eddy of equal scale.

According to Levich [13], eddy velocity and eddy scale are related by

$$v_{\lambda} = (\epsilon \lambda)^{\frac{1}{3}} \quad (120)$$

for  $\lambda > \lambda_0$ ,  $\lambda_0$  being the microscale of the turbulence as defined by Equation 81. The characteristic diffusion time is estimated by

$$\tau_0 = \frac{\lambda}{v_{\lambda}} \quad (121)$$

Substituting  $R_F$  for  $\lambda$ , and allowing for Equations 4 and 119, the characteristic diffusion time becomes

$$\tau_0 = \left( \frac{R_F^2}{G^2 \nu} \right) \quad (122)$$

Diffusion times calculated from Equation 122 are given in Table V. Also given in this table is the microscale,  $\lambda_0$ , which seems to be of the same magnitude as  $R_F$ , not quite as required for Equation 119 to hold. However, for calculating relative values of  $D_{eff}$  this deficiency may be disregarded.

TABLE V  
CHARACTERISTIC DIFFUSION TIMES

| G<br>sec <sup>-1</sup> | R <sub>F</sub><br>microns | λ <sub>0</sub><br>microns | τ <sub>0</sub><br>sec   |
|------------------------|---------------------------|---------------------------|-------------------------|
| 30                     | 270                       | 180                       | 4.3 x 10 <sup>-2</sup>  |
| 120                    | 65                        | 90                        | 6.65 x 10 <sup>-3</sup> |
| 240                    | 25                        | 64                        | 2.21 x 10 <sup>-3</sup> |

The frequency range of most significance in the flocculation process has a lower limit  $f_0$  which is given by

$$f_0 = \frac{1}{2\pi} v^{\frac{1}{3}} G^{\frac{2}{3}} R_F^{-\frac{2}{3}} \quad (123)$$

by virtue of Equations 4 and 47.

An estimated value for  $D_{eff}$  was obtained through replacing the integral of Equation 77 by

$$D_{eff} = E(f_0) \frac{Af_0^{\frac{3}{2}}}{2 R_1 + Af_0^{-\frac{3}{2}}} \cdot \tau_0 \cdot \Delta f \quad (124)$$

where  $E(f_0)$  is the spectral density at  $f_0$ , taken from Figure 53;  $\tau_0$  is the characteristic diffusion time, which is taken from Table V, in place of the diffusivity filter function, and  $\Delta f$  is the frequency range over which  $E(f)$  decreases by a factor of 10.  $D_{eff}$  obtained in this manner can be regarded as a rough estimate, accurate to within an order of magnitude at the most. The calculations and results are shown in Table VI. Comparing the values of  $D_{eff}$  obtained by the two different and independent methods (Tables IV and VI), it is seen that for the stake and stator paddle the correlation is remarkably good. For the turbine paddle the corresponding values agree to within an order of magnitude at  $G$  of 30 and 120  $\text{sec}^{-1}$ , and to within two orders of magnitude at  $G$  of 240  $\text{sec}^{-1}$ .

The somewhat arbitrary procedure by which  $\Delta f$  was calculated was selected so that the calculated values of  $D_{eff}$  will come as close as possible to the values determined from the flocculation equation. The uncertainties regarding the diffusivity filter function,  $f_0$ , and  $\Delta f$  make any accurate determination of  $D_{eff}$  impossible. However, both calculation methods predicted a decrease in  $D_{eff}$  with  $G$  for each paddle type, with higher values for the stake and stator paddle at any given  $G$ . Hence the qualitative nature of the foregoing analysis can be regarded as further support of the proposed model.

The decrease in effective diffusivity with  $G$ , may look contradictory to one's intuitive prediction. In this regard, it should be noticed that this result stems from the assumption that floc size is inversely proportional to the power input, an assumption which was valid for the system under investigation. However, this relationship reflects a physical property of the flocs, namely the floc strength which can be modified by chemical means, e.g., the use of polyelectrolytes. If the floc size could be maintained at a constant value independent of  $G$ , or if the dependence could be expressed as in Equation 75 with  $K_3 \ll 1$ , then  $D_{eff}$  would increase with  $G$  as expected intuitively.

TABLE VI  
CALCULATION OF  $D_{eff}$  FROM TURBULENCE ENERGY SPECTRUM

| G<br>sec <sup>-1</sup> | $R_F$<br>microns | A<br>cm sec <sup>-2</sup> | $f_o$<br>sec <sup>-1</sup> | $e_f^{(1)}$ | $\Delta f$<br>sec <sup>-1</sup> |         | $E(f_o)$<br>cm <sup>2</sup> sec <sup>-1</sup> |         | $D_{eff}$<br>cm <sup>2</sup> sec <sup>-1</sup> |                       |
|------------------------|------------------|---------------------------|----------------------------|-------------|---------------------------------|---------|-----------------------------------------------|---------|------------------------------------------------|-----------------------|
|                        |                  |                           |                            |             | Stake<br>and<br>Stator          | Turbine | Stake<br>and<br>Stator                        | Turbine | Stake<br>and<br>Stator                         | Turbine               |
| 30                     | 170              | 0.19                      | 5.5                        | 0.975       | 6.5                             | 5.5     | 0.25                                          | 0.005   | $6.8 \times 10^{-2}$                           | $1.15 \times 10^{-3}$ |
| 120                    | 65               | 0.76                      | 24                         | 0.955       | 30                              | 30      | 0.05                                          | 0.01    | $9.5 \times 10^{-3}$                           | $1.90 \times 10^{-3}$ |
| 240                    | 25               | 1.52                      | 72                         | 0.892       | 90                              | 40      | 0.015                                         | 0.0015  | $2.7 \times 10^{-3}$                           | $1.18 \times 10^{-5}$ |

$$(1) \quad e_f = \frac{Af^{-\frac{3}{2}}}{2R_1 + Af^{-\frac{3}{2}}}$$



The value of the constant  $K_s$  can be estimated from the approximate values of  $K_p$  and the product  $K_s K_p$ . From Figure 45 the values of  $K_p$  were found to be  $4.5 \times 10^{-2}$  and  $2.3 \times 10^{-2}$  cm<sup>2</sup>/sec for the stake and stator and the turbine paddles, respectively. With  $K_s K_p$  values of 6.85 and 5.18, the corresponding values of  $K_s$  are 152 and 226 sec/cm<sup>2</sup>.

Hence the energy spectrum constants are approximately 180 sec/cm<sup>2</sup> for both paddle types. The smaller  $K_s$  for the stake and stator paddle means that a smaller fraction of the total  $u^2$  is effective in the flocculation process. Or, in other words, the relative contribution to  $u^2$  from the significant frequencies range is smaller. The energy spectra in Figure 53 do indicate this.

A theoretical expression for  $D_{eff}$  can be derived for the special case when  $f_0$  is within the universal equilibrium range and the diffusion time  $\tau_0$  is short enough so that the diffusivity filter function can be approximated by  $\tau_0$ . Under these conditions the entrainment factor  $e_f$  is also very close to unity and the effective diffusion coefficient becomes

$$D_{eff} = e_f \tau_0 K \epsilon^{\frac{2}{3}} \int_{f_0}^{\infty} f^{-\frac{5}{3}} df \quad (124a)$$

by virtue of Equations 77 and 83.

Performing the integration and allowing for Equations 4 and 123 the result is

$$D_{eff} = \frac{3}{2} (2\pi)^{\frac{2}{3}} K \tau_0 e_f \nu^{\frac{4}{9}} R_F^{\frac{4}{9}} G^{\frac{8}{9}} \quad (124b)$$

No attempt has been made to calculate numerical values of  $D_{eff}$  from this equation. However, it is of interest to notice that  $D_{eff}$ , and hence also the rate of collisions, varies almost linearly with  $G$  in accordance with the independent analytical results obtained by Camp and Stein (Equation 5) and Saffman and Turner (Equation 7).

#### APPLICATION OF THE THEORY TO MULTICOMPARTMENT SYSTEMS

The performance parameter of a multicompartment system can be deduced from Equation 88. Thus, for a system composed of  $m$  equal compartments in series with an overall residence time  $T$ , the performance parameter is given by



$$\frac{n_1^0}{n_1^m} = \prod_{i=1}^m \left[ \frac{1 + K_F^i K_P^i K_S^i n_1^0 G^i \frac{T}{m}}{1 + K_B^i \frac{n_1^0}{n_1^{i-1}} G^{i^2} \frac{T}{m}} \right] \quad (125)$$

Single compartment data can be used to predict the performance of a multicompartment system if the constants  $K_F$ ,  $K_P$ ,  $K_S$ , and  $K_B$  are assumed to be the same in each of the compartments in the series. Under these conditions Equation 125 reduces to

$$\frac{n_1^0}{n_1^m} = \frac{\left( 1 + K_A G \frac{T}{m} \right)^m}{1 + K_B G^2 \frac{T}{m} \sum_{i=0}^{m-1} \left( 1 + K_A G \frac{T}{m} \right)^i} \quad (126)$$

where

$$K_A = K_F K_P K_S n_1^0 \quad (127)$$

Experimental values of the performance parameter of a multicompartment system are compared with the corresponding theoretical values in Figures 58 through 61. The theoretical curves were plotted using Equation 126 with  $K_A$  and  $K_B$  determined from single compartment data, presented in previous sections. Thus, from Figure 57, the following values for  $K_A$  and  $K_B$  were obtained;

$$K_A = \frac{a}{c} = 5.14 \times 10^{-5}$$

$$K_B = \frac{b}{T} = 1.08 \times 10^{-7} \text{ sec} \quad .$$

These values of  $K_A$  and  $K_B$  were used in calculating the theoretical performance parameters of two- and four-compartment systems as shown in Figures 58 through 61. It should be noticed that each of these figures was plotted for a fixed  $T/m$ . Hence the difference in performance is due both to the difference in overall residence time and the compartmentalization effect. Comparing the experimental data with the theoretical curves, a general agreement is apparent. However, in absolute values, the experimental data differ from the theoretical curves. In the low- $G$  region the experimental performance seems to agree with the predicted whereas in the high- $G$  region the experimental performance is higher. This suggests that the constants  $K_A$  and  $K_B$  do vary from one compartment

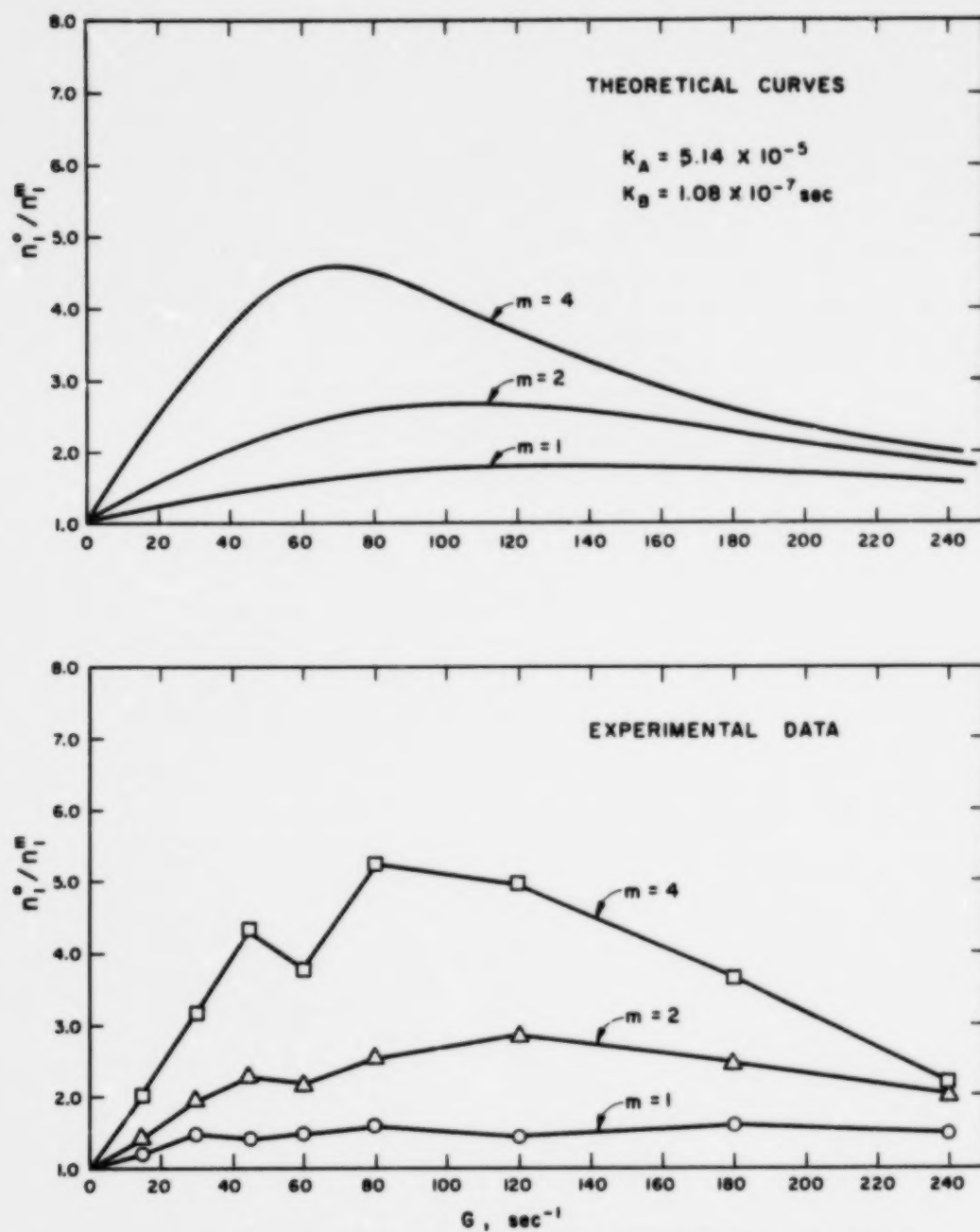


FIGURE 58. PERFORMANCE OF MULTICOMPARTMENT SYSTEMS

$$\frac{T}{m} = 4 \text{ min.}$$

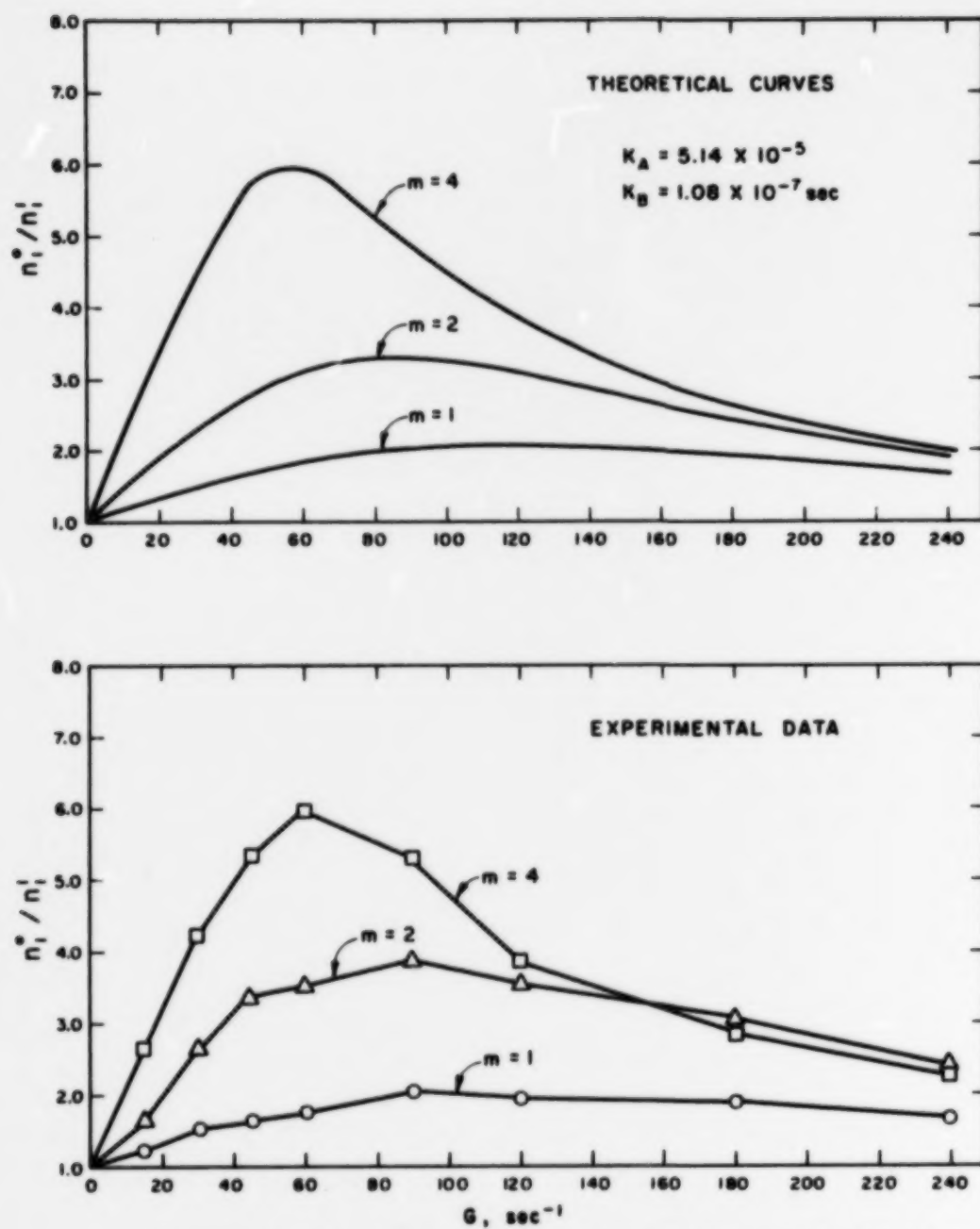


FIGURE 59. PERFORMANCE OF MULTICOMPARTMENT SYSTEMS

$$\frac{T}{\bar{m}} = 6 \text{ min.}$$

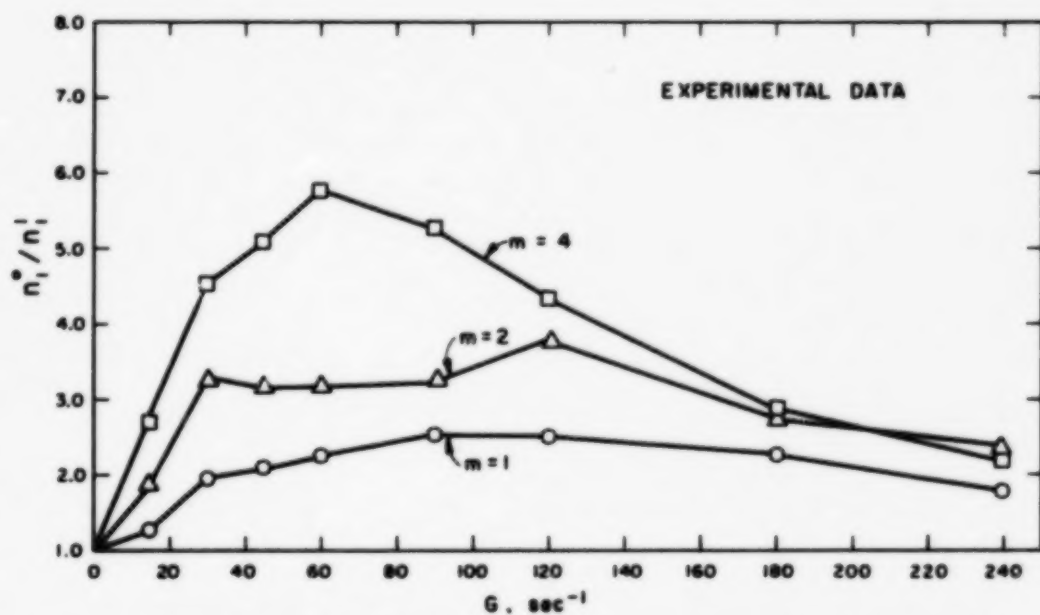
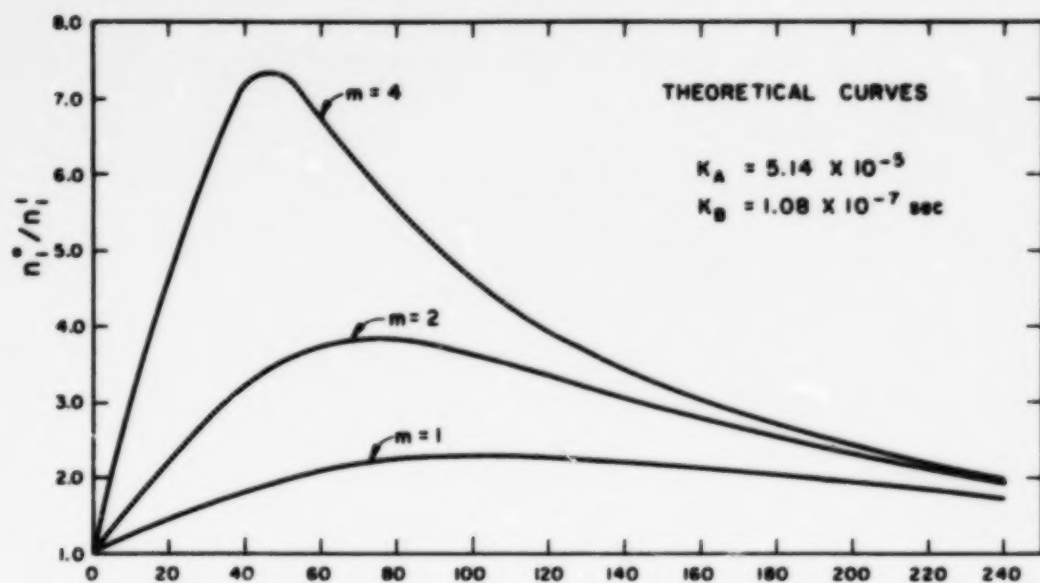


FIGURE 60. PERFORMANCE OF MULTICOMPARTMENT SYSTEMS

$$\frac{T}{m} = 8 \text{ min.}$$

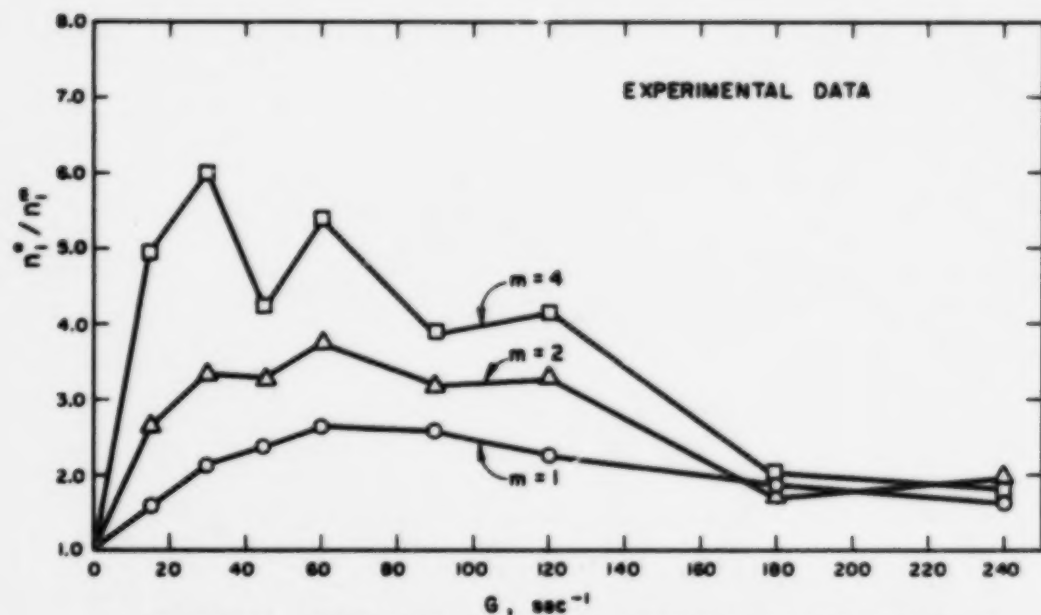
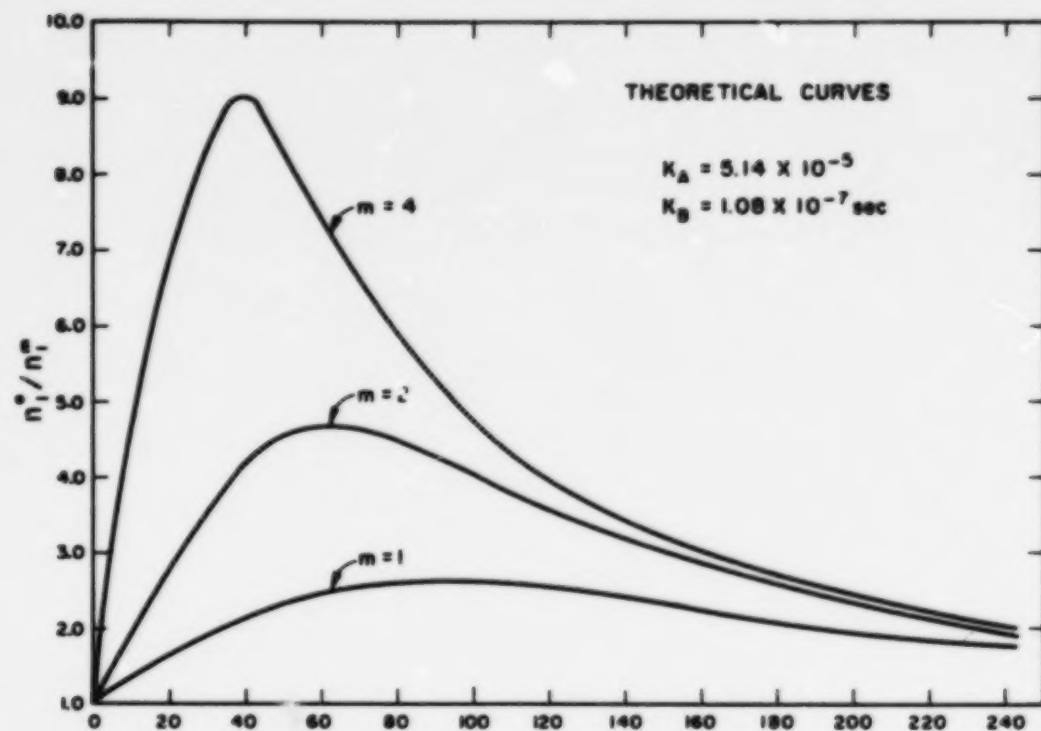


FIGURE 61. PERFORMANCE OF MULTICOMPARTMENT SYSTEMS

$$\frac{T}{m} = 12 \text{ min.}$$



to the next. Moreover, the indication is that both  $K_A$  and  $K_B$  decrease in successive compartments. As there is no reason to believe that the turbulence characterizing constants change with compartments, it can be concluded that  $K_P$  and  $B$  are the varying constants. Specifically, these constants decrease with increasing  $m$ . In view of the previous discussion regarding the dependence of the aggregation-collision ratio,  $\alpha$ , on floc properties, it is expected that flocs of more regular shape and more compact structure will tend to have a lower value of  $\alpha$ . In addition, flocs in the tail end compartments will presumably be stronger as they have been exposed to shearing stresses for longer periods and survived. Thus, as  $K_P$  is directly proportional to  $\alpha$  and  $B$  inversely proportional to floc strength, the observed difference between the experimental data and the theoretical prediction has a rational explanation.

The net effect of compartmentalization, as predicted by Equation 126, is shown in Figure 62 where the performance parameter of a single reactor is compared to a multicompartment system of equal overall residence time. As is evident from the figure, the theory predicts an increase in performance with increasing  $m$ . The effect is greatest in the medium- $G$  range, becoming insignificant at very high  $G$ . These results are in agreement with the experimental findings shown in Figures 15 through 18.

#### IMPLICATION OF RESULTS IN PROCESS DESIGN

In designing a flocculation process unit, the engineer should make several decisions regarding the most suitable chemicals to be employed and the physical characteristics of the unit in which the process is to be carried out while at the same time recognizing that flocculation is an element of the total clarification system that includes sedimentation and filtration. Some indication of the most appropriate coagulants and their concentrations can be obtained through preliminary jar tests, but if the design is to be accompanied with assurance of success, pilot plant tests are very desirable and may be accomplished with equipment of the type employed in this investigation. Such tests will aid the designer in determining the size and configuration, especially compartmentalization, of the flocculation basin as well as the geometry and power requirements of the stirring device. This section of the report will deal with design criteria for the flocculation reactor and the application of the data obtained in this investigation to selecting these criteria.

The currently accepted design criteria for ascertaining the reactor volume and stirring system are based on the concept that performance in a linear function of  $G$  and  $T$ . As these two are generally accepted as independent, their dimensionless product is regarded as the most important parameter. Based on practical experience as well as laboratory investigations, recommended values of  $GT$  range from  $2 \times 10^4$  to  $2 \times 10^5$  [48]. To supplement those design criteria, the individual values of  $T$  and  $G$  are also restricted to recommended ranges which are 10 to 60 minutes and 10 to 75  $\text{sec}^{-1}$ , respectively [49]. According to



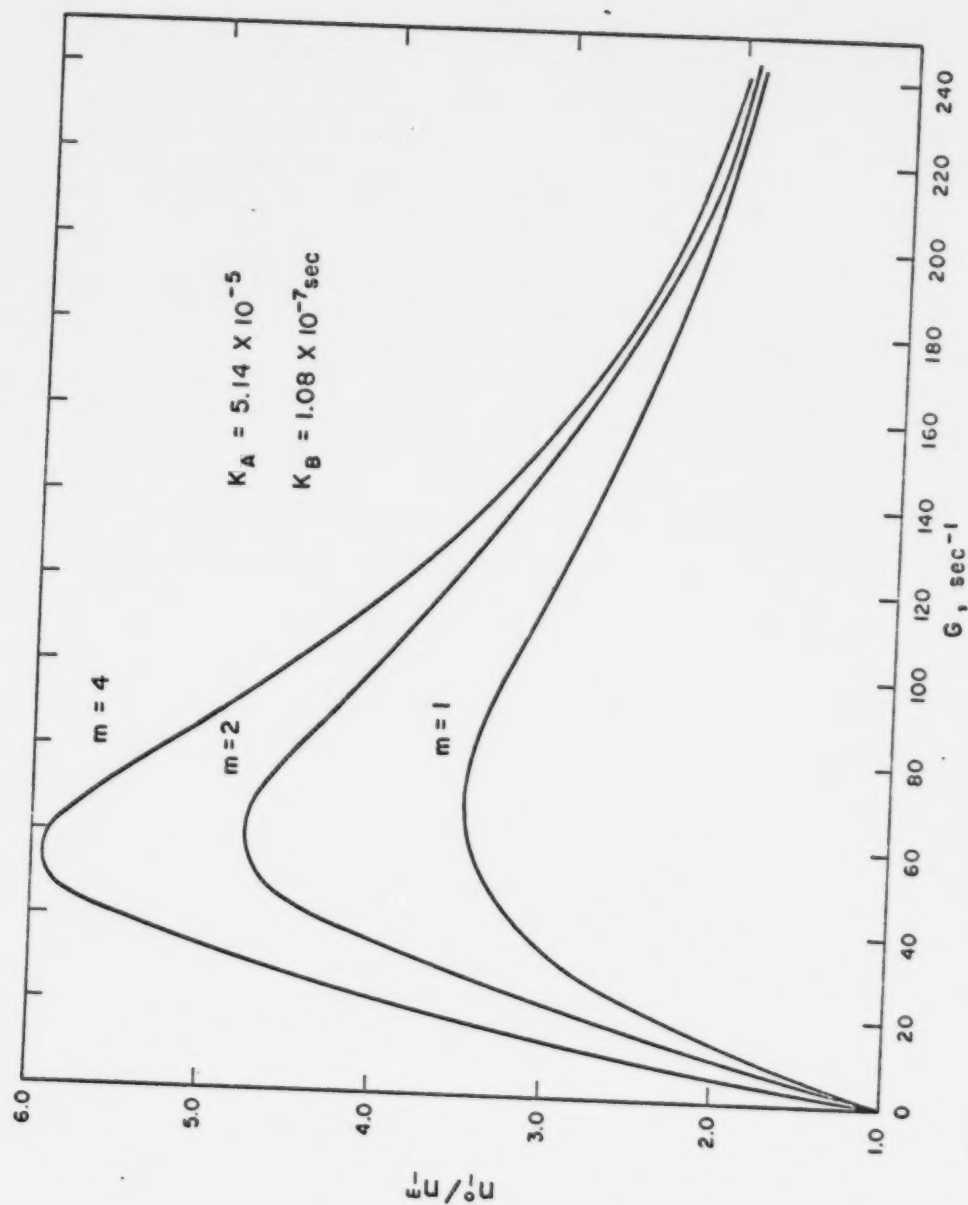


FIGURE 62. EFFECT OF COMPARTMENTALIZATION ON FLOCCULATION PERFORMANCE.  $T = 24 \text{ min.}$

the theory advanced in this study, the relationship between these parameters and the flocculation performance is given by

$$\frac{n_1^0}{n_1^1} = \frac{1 + K_F K_S K_P n_1^0 T G}{1 + K_B T G^2} \quad (85)$$

for a single reactor. This equation shows that the linear dependence of the performance parameter on  $GT$  is limited to low values of  $GT$ . The region of linearity depends mainly on the magnitude of the breakup constant,  $K_B$ , which is inversely proportional to floc strength. The inadequacy of  $GT$  as a sole design criterion is illustrated by Figure 63. The curves in this figure were plotted from Equation 85 using the empirical constants calculated previously for the stake and stator paddle arrangement. As can be seen from the figure, the performance parameter of a flocculation reactor depends on both  $G$  and  $T$  but not on the product  $GT$ . It should be noted that the optimum  $GT$  values predicted by the proposed theory are within the recommended range. However, this theory has the advantage that it provides the mathematical relationship by which  $G$  and  $T$  can be selected for any desired performance, provided the constants are known or can be estimated.

The interrelation among  $G$ ,  $T$ , and the performance parameter as expressed in Equation 85 is also shown in Figure 64. Some interesting conclusions may be drawn from the last two figures. First, it is evident that for any desired performance there is a minimum residence time below which this performance cannot be achieved regardless of  $G$ . It also shows that at very long residence times, small variations in  $G$  result in relatively large changes in performance. Finally it shows that when  $G$  is larger than some optimum value, an increase in  $G$  has an adverse effect on performance.

The use of  $G$  and  $T$  as the only parameters characterizing the physical properties of a flocculation system has two additional deficiencies. First, it is evident that residence time distribution has a decisive effect on the overall system performance. This is illustrated in the experimental results shown in Figures 15 through 18 and by Figure 62. In the theory presented herein, the effect of residence time distribution is given a mathematical expression in the form of Equation 126. Experimental data indicate, however, that this equation underestimates the performance of multicompartiment systems operating in the high  $G$  range. Hence, its usefulness for practical purposes is somewhat limited.

The second deficiency stems from the fact that the observed difference in performance between the two paddle types at identical  $G$  values provides sufficient evidence that  $G$  alone is not adequate for characterizing the agitation conditions. The proposed theory identifies  $u^2$  and its spectrum as the only energy parameters directly affecting the flocculation process. The reason why  $G$  was introduced into the

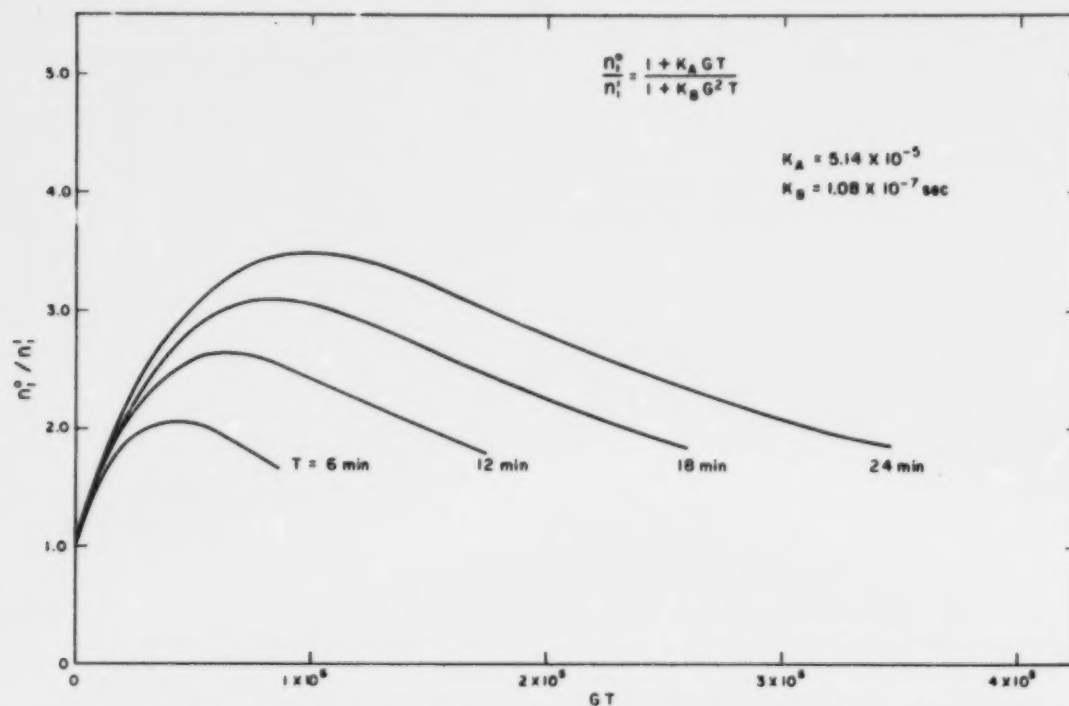


FIGURE 63. EFFECT OF GT ON PERFORMANCE OF A SINGLE REACTOR

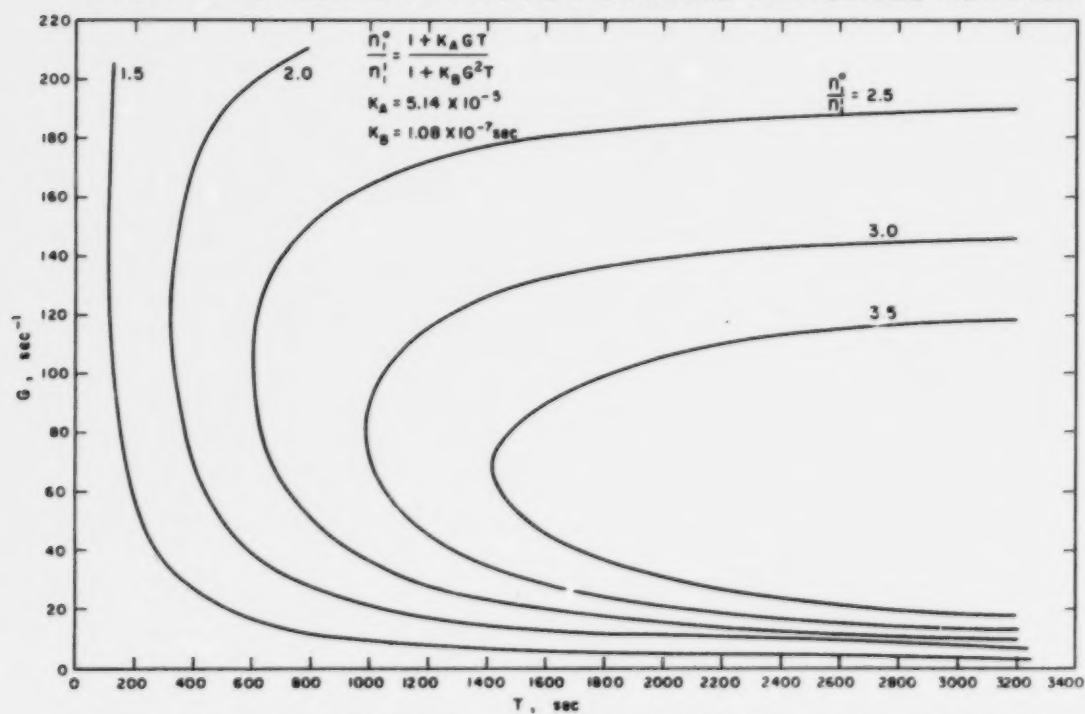


FIGURE 64. RELATIONSHIP OF PERFORMANCE TO G AND T FOR A SINGLE REACTOR

flocculation equation was the relative ease in which it is estimated and its acceptance among sanitary engineers.

There is one special case in which  $G$  is an adequate parameter for describing the effect of the stirring mechanism on flocculation. This occurs when the size of flocs and primary particles involved in the process are much smaller than the microscale of the turbulence field. Under these conditions, the eddies responsible for particles collisions are within the universal equilibrium range in which the energy spectrum is a function of the energy dissipation alone, thus uniquely proportional to  $G$ . As can be seen from Table V, in the power input range encountered in water treatment practice, the floc sizes are not much smaller than the microscale of the turbulence. Hence, in general the flocculation process is affected to some extent by eddies which are larger than the ones in the universal equilibrium region. The energy spectrum in this region depends on the properties of the turbulence generating mechanism, i.e., on the stirring device. This fact was expressed in the proposed theory by introducing the two constants  $K_p$  and  $K_s$ . The former is equal to  $u^2/G$  and is a property of the stirring mechanism only. The latter represents the relative contribution to  $u^2$  from eddies of size similar to the collision radius and hence is a property of both the energy spectrum and the floc size.

The flocculation equation derived in this study can be put to practical use for process unit design provided the constants are determined from pilot plant studies similar to those performed in the present study. However, for practical purposes, only  $K_p$  and the product  $K_p K_s K_p$  are of interest. Where pilot plant studies are not practical, or for preliminary design, these constants can be estimated from the empirical values determined in the current study or from performance data of existing plants. Using the constants of the present study, the flocculation equation for a single reactor with stake and stator type of paddles becomes

$$\frac{n_1^0}{n_1^1} = \frac{1 + 6.85 \times 10^{-11} n_1^0 G T}{1 + 1.08 \times 10^{-7} G^2 T} \quad (128)$$

The applicability of this equation to other flocculation systems was tested by comparing the experimental data obtained by Harris [34] with those predicted for his system. Harris carried out his experiments in a cylindrical reactor about 3.5 times as large as the one used in this study. His primary particles suspension was made of about 25 mg/l kaolin clay, with size distribution similar to that of the currently used clay. The raw water was flocculated in a continuous flow system with 25 mg/l of commercial alum. The stirring device consisted of rotating and stationary paddles resembling the stake and stator arrangement of the present study. Figure 65 shows the experimental results reported by Harris and the predicted values calculated from Equation 128. The relatively good agreement between the two is not surprising

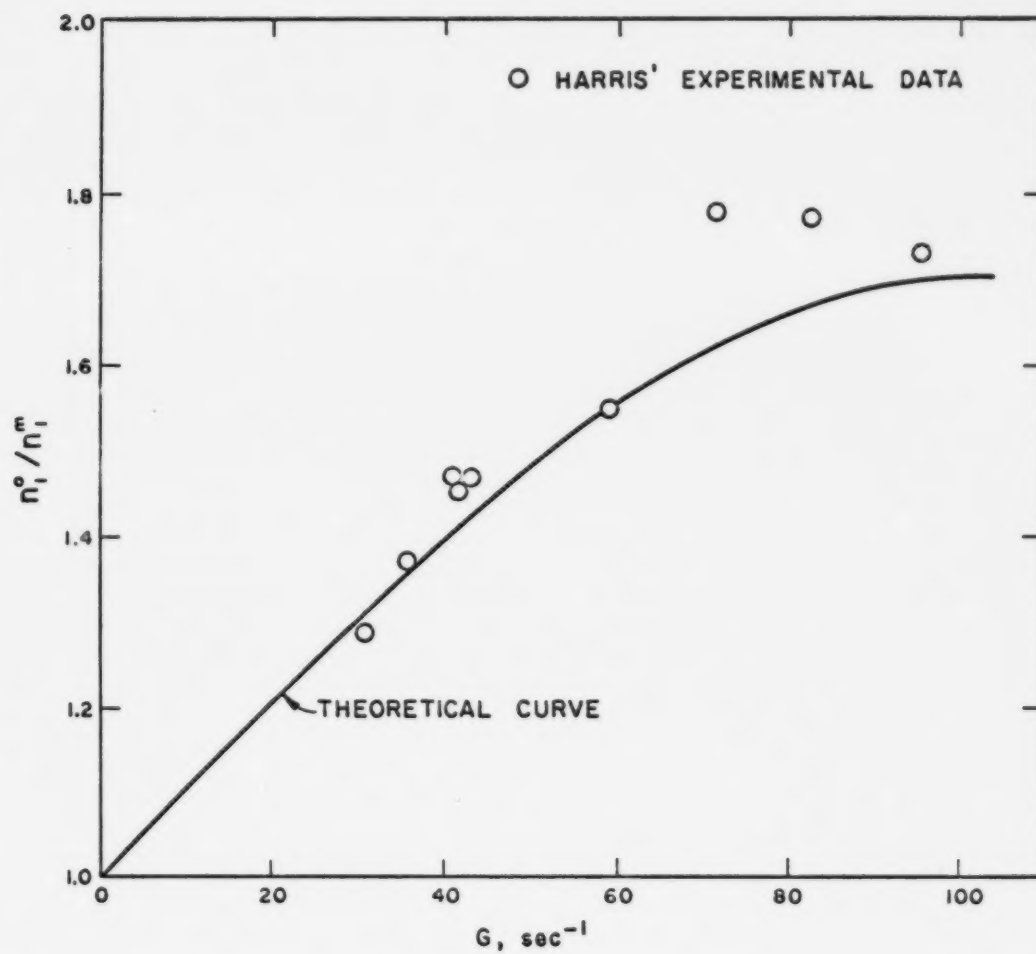


FIGURE 65. APPLICABILITY OF THEORY TO HARRIS' FLOCCULATION SYSTEM



in view of the fact that the waters were essentially identical. It suggests, however, that pilot plant data may be extrapolated to larger units if the studies are carried out with similar suspensions and chemical dosages. An example of this extrapolation is given in Appendix II in which the pilot plant employed herein was operated in parallel with the full-sized flocculators of the East Bay Municipal Utility District's Upper San Leandro Plant. Nearly identical performances were observed with both units.

The effect of chemical factors on the flocculation rate is a very complex one and is of greatest importance from a practical viewpoint. Three of the constants in Equation 85 are directly affected by the chemical properties of the flocculating system. Quantitative relationships between these constants and the chemical parameters are unknown, but some qualitative predictions can be made. The flocculation constant,  $K_F$ , is very sensitive to chemical changes, mainly because the aggregation-collision ratio,  $\alpha$ , depends on the degree of instability of the suspension and the intensity of the interparticle attraction forces, both of which are affected by chemical factors. The breakup constant,  $K_B$ , which was defined as

$$K_B = B K_P^2 R_1^3 \frac{K_1}{K_2} \quad (87)$$

depends primarily on floc strength. An increase in floc strength will result in less erosion of primary particles from the floc surface, thus decreasing  $B$ . It will also result in a tendency to increase floc size which will be indicated by a larger  $K_2$  (cf., Equation 76). In addition to the decrease in  $K_B$ , the increase in floc size will affect the flocculation constant  $K_F$  by increasing  $K_S$ . This is because a larger portion of the turbulence spectrum will be effective in flocculation. The result is that floc strength is probably the most significant factor through which chemical factors may affect the flocculation rate. This explains the great potential of polyelectrolytes in improving the performance of flocculation units and permitting the use of greater values of  $G$ .

The effect of the geometrical properties of the stirring device on flocculation rate is expressed by the paddle performance coefficient,  $K_P$ . The stake and stator paddle arrangement, which proved to be better than the turbine paddle, had a larger  $K_P$  value. This could not be attributed to a better spatial distribution of the power input. The most probable reason for the better performance and the higher  $u^2$  values obtained with this paddle arrangement is the combination of rotating and stationary elements. The advantages of stationary baffles in a mixed container are well known and documented.

The relationship of performance to  $G$ ,  $T$ , and  $m$  is shown in Figure 66. This figure illustrates one of the more significant conclusions of this study from the practical viewpoint, namely, the effect of compartmentalization. The addition of a few dividing walls to a single compartment reactor, which adds very little to the construction



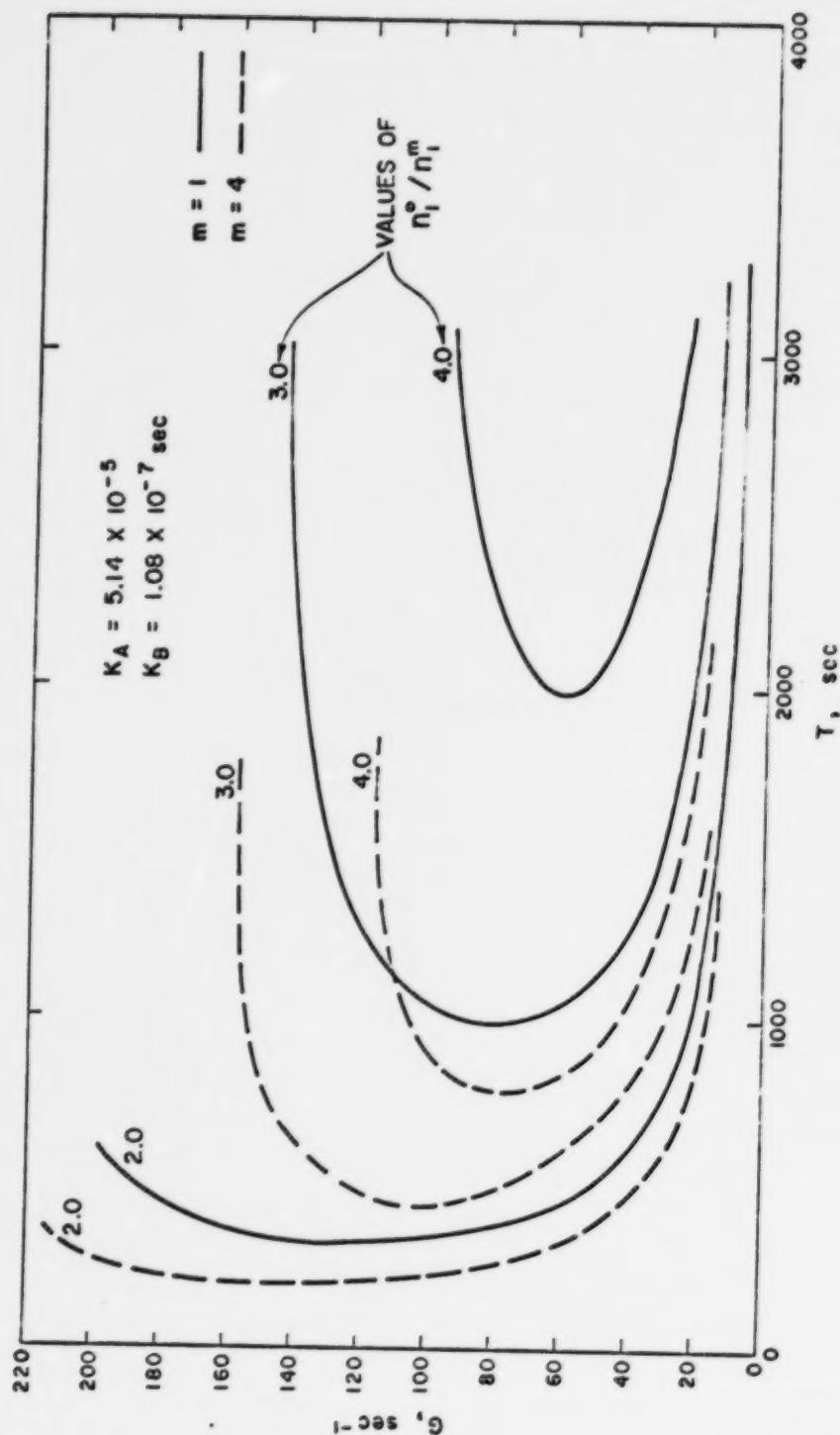


FIGURE 66. RELATIONSHIP OF PERFORMANCE TO G, T, and m

costs of the plant, will generally have a considerable effect on the performance. The effect of compartmentalization is greatest at the high performance range. For example, if the desired performance is 4.0, the four-compartment system requires 13 minutes residence time with a  $G$  of  $75 \text{ sec}^{-1}$  as compared to 33 min and  $60 \text{ sec}^{-1}$  in a single reactor system.

## VII. SUMMARY AND CONCLUSIONS

The broad aims of this investigation were to provide a better insight into the flocculation process, the various physical parameters involved, and their impact on the performance of the process. It was hoped that the findings of this study could ultimately be used by practicing sanitary engineers to improve the design and operation of flocculation units in water and waste water treatment plants. The need for a better understanding of the process stems from its frequent application in water treatment and from the empirical and subjective nature of the design criteria employed by practitioners in the field.

The theoretical part of the investigation was devoted to developing a rational model for the collision mechanism between suspended particles in a turbulent environment. This model was predicated on the hypothesis that the random motion of suspended particles in a turbulent fluid may be characterized by a diffusion coefficient. The problem was then reduced to exploring the relationship between this coefficient and the hydrodynamic characteristics of the turbulence. For this purpose the turbulence energy spectrum proved to be very useful. With an expression for the effective diffusivity at hand, it was possible to derive an expression for the rate of collisions between the suspended particles. The method used for this derivation was similar to that employed for determining collision frequency between suspended particles subjected to Brownian motion. The expression thus derived showed that the collision frequency was a function of both the turbulence energy spectrum and particle size. The rate at which suspended particles are removed from a flocculating suspension due to their successful collisions with other particles was then related to the collision frequency by introducing the aggregation-collision ratio. A rate equation describing the change in concentration of primary particles was developed by considering their simultaneous removal by aggregation and reformation by breakup. A simplified breakup model was proposed, implying that the only mode by which primary particles are formed is through erosion from the surface of preformed flocs. The differential rate equation was then solved for a continuous flow completely mixed system. The resulting equation expressed the relation between the performance parameter of a single reactor and the physical properties of the system. To facilitate the use of the equation for practical purposes the following assumptions were made:

1. The effective diffusivity is directly proportional to an energy spectrum coefficient, to the mean square of the turbulent velocity fluctuations, and to the square of the floc radius.
2. The number concentration of flocs was assumed to be proportional to the initial concentration of primary particles (cf., Equations 74 and 75). Here it should be appreciated that this assumption is not entirely valid

and that the floc number concentration is dependent on many other factors including its strength, the coagulant dose and pH, and of course the turbulence characteristics (cf., Figures 24 and 25). An alternate approach has been suggested that avoids to some degree this problem.

3. The floc radius is inversely proportional to the mean square velocity fluctuations.
4. The mean square velocity fluctuations are directly proportional to the rms velocity gradient.

The last two assumptions were substantiated by the experimental results of the study.

With these assumptions, a working equation for a single reactor flocculation system was derived and the equation was then generalized to accommodate compartmentalization.

The second part of the investigation was devoted to experimental verification of the proposed theory. The experimental work consisted of the following:

1. Flocculation tests in a single reactor with two types of stirring mechanisms.
2. Flocculation tests in multicompartment systems with one type of stirring mechanism.
3. Turbulence measurements in stirred reactors with both types of stirring mechanisms.
4. Residence time distribution studies.
5. Floc size measurements.

All flocculation tests were carried out in a laboratory scale flocculator specially designed for the study. The performance of the system was determined by comparing the number concentration of primary particles in the feed water to that in the settled effluent. Performance was measured over a wide range of stirring energies and retention times. The single reactor studies showed that the theoretical equation adequately described the experimental results. This was considered to be a verification of the general validity of the proposed model. The results also showed that one type of stirring device - the stake and stator - was consistently better than the other. This was somewhat in contradiction to the generally accepted assumption that the power input is the only parameter of a stirring mechanism which affects the flocculation process.

Residence time distribution studies were performed with both stirring devices. These were accomplished by a common tracing test.

The results showed that both stirring devices produced essentially the same residence time distributions at all energy inputs. This result ruled out residence time distribution as a possible source of the difference in performance between the two stirrers.

Turbulence measurements were carried out using a hot-film anemometer, a rms voltmeter, and a high-speed digital recorder. With the anemometer-voltmeter assembly it was possible to measure the spatial distribution of the mean square velocity fluctuations at various power inputs. The results of these measurements showed that the mean square velocity fluctuations are directly proportional to the rms velocity gradient (G). The proportionality constant, which was referred to as the "paddle performance coefficient," was found to be larger for that stirring device which gave better flocculation performance. With a high-speed digital recorder, the time variation of the turbulence velocity was recorded on magnetic tapes and then processed on the IBM 7094 computer. The analysis yielded the turbulence energy spectrums produced by the two stirring devices at various power inputs. From these, a rough estimate of the effective diffusion coefficient was calculated and found to be approximately comparable to the value determined from the flocculation tests.

The dependence of floc size on power input was explored by photographing undisturbed floc samples under a microscope. The results showed that floc size is inversely proportional to the rms velocity gradient.

A series of flocculation tests in a multicompartiment system was also performed and the results were compared to those predicted by the theory. The predicted performance was calculated by assuming that all the constants in the flocculation equation were invariant throughout the system. Under these assumptions it was found that for low power inputs the theory adequately estimates the performance of multicompartiment systems while for high power inputs the performance is underestimated.

The results of the theoretical and experimental aspects of this investigation may be summarized in the following conclusions:

1. The physical parameters directly affecting the performance of a flocculation reactor are the residence time, its distribution, the total power input, and the energy spectrum of the turbulence field.
2. For a given stirring mechanism the effect of the turbulence energy spectrum may be expressed by the rms velocity gradient, or the square root of the power input, a paddle performance coefficient, and an energy spectrum coefficient.
3. For a given residence time the performance increases almost linearly with the rms velocity gradient until a maximum value is reached beyond which any further increase results in a decrease in performance.



4. For any desired performance there is a minimum residence time requirement. The rms velocity gradient associated with this retention period is an optimal value in the sense that any other value will result in lower performance.
5. Compartmentalization of a continuous flow system exerts a considerable influence on the flocculation performance. Systems with equal overall retention periods will perform better as the degree of compartmentalization increases.
6. There are strong indications that pilot plant data may be extrapolated and used for full scale plant design, provided the chemical conditions are the same in both systems.
7. The chemical conditions prevailing in a flocculation system affect the kinetics of the process mainly through floc strength and the aggregation-collision ratio. Floc strength is of major importance as it influences both the rate of aggregation and the rate of breakup.

The above listed conclusions provide useful guidance for design and operation of flocculation units in water purification plants. Some basic questions still remain open and will require additional study. Among these, the most important from the practical point of view are the quantitative relationships between; 1) the constants of the flocculation equation and the appropriate chemical parameters, and 2) a quantitative expression relating the performance coefficient of a stirring mechanism to its geometry.

By developing a rate equation in which all, or nearly all, of the physical parameters affecting flocculation are incorporated, a rational and practical basis has been provided on which to conduct studies of the chemical aspects of coagulation-flocculation. It is believed that the single continuous stirred tank reactor affords an excellent means of investigating the chemical parameters and, together with the rate equation developed herein, of expressing the results in terms meaningful to the water and waste water industry.



APPENDICES

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APPENDIX A  
LIST OF SYMBOLS

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## LIST OF SYMBOLS (Continued)

| Symbol                                                                             | Definition                                                                               |
|------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| F                                                                                  | Force on immersed body; residence time distribution function; smoothed Fourier transform |
| F'                                                                                 | Unsmoothed Fourier transform                                                             |
| F <sub>A</sub>                                                                     | Attenuation factor                                                                       |
| f                                                                                  | Frequency                                                                                |
| G                                                                                  | rms velocity gradient                                                                    |
| H                                                                                  | Frequency of collisions in turbulent flow                                                |
| h                                                                                  | Integer                                                                                  |
| I                                                                                  | Frequency of collisions in Brownian motion                                               |
| J                                                                                  | Frequency of collisions in Laminar flow                                                  |
| K <sub>1</sub> , K <sub>2</sub> , K <sub>3</sub> , K <sub>4</sub> , K <sub>5</sub> | Constants                                                                                |
| K <sub>B</sub>                                                                     | Breakup constant                                                                         |
| K <sub>F</sub>                                                                     | Flocculation constant                                                                    |
| K <sub>P</sub>                                                                     | Paddle performance coefficient                                                           |
| K <sub>S</sub>                                                                     | Energy spectrum constant                                                                 |
| k                                                                                  | Boltzman's constant                                                                      |
| M                                                                                  | Number of data points in correlation array                                               |
| m                                                                                  | Number of equal compartments in a series; dead space fraction                            |
| N                                                                                  | Paddle rotation speed; number of observations                                            |
| n                                                                                  | Number concentration; constant in King's equation                                        |
| P                                                                                  | Power                                                                                    |
| p                                                                                  | Plug-flow fraction                                                                       |
| q                                                                                  | Power supply to hot-film sensor                                                          |

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## LIST OF SYMBOLS (Continued)

| Symbol    | Definition                                                |
|-----------|-----------------------------------------------------------|
| $R$       | Radius; collision radius                                  |
| $R_e$     | Reynold's number                                          |
| $R_E$     | Eulerian time-correlation coefficient                     |
| $R_L$     | Lagrangian time-correlation coefficient                   |
| $R_p$     | Particle time-correlation coefficient                     |
| $r$       | Coordinate in polar system                                |
| $S$       | Standard deviation; slope of linear trend                 |
| $T$       | Mean residence time; absolute temperature                 |
| $t$       | Time                                                      |
| $U$       | Instantaneous velocity in the x-direction                 |
| $U_{eff}$ | Effective velocity past hot-film sensor                   |
| $u$       | Turbulent velocity fluctuation in the x-direction         |
| $V$       | Instantaneous velocity in the y-direction; reactor volume |
| $V_s$     | Suppressing voltage                                       |
| $v$       | Turbulent velocity fluctuation in the y-direction         |
| $v_p$     | Velocity of particle                                      |
| $v_r$     | Velocity of particle relative to fluid                    |
| $W$       | Instantaneous velocity in the z-direction                 |
| $X$       | Value of individual observation                           |
| $x$       | Coordinate in cartesian system                            |
| $y$       | Coordinate in cartesian system                            |
| $Z$       | Output data from digital recorder                         |

## LIST OF SYMBOLS (Continued)

| Symbol      | Definition                                       |
|-------------|--------------------------------------------------|
| $z$         | Coordinate in cartesian system                   |
| $\alpha$    | Aggregation-collision ratio                      |
| $\beta$     | Constant; particle inertia parameter             |
| $\gamma$    | Ratio of fluid to particle density               |
| $\epsilon$  | Power input or power dissipation per unit mass   |
| $\delta$    | Particle size distribution function              |
| $\lambda$   | Eddy scale                                       |
| $\lambda_0$ | Microscale of turbulence                         |
| $\mu$       | Fluid viscosity                                  |
| $\nu$       | Fluid kinematic viscosity                        |
| $\rho$      | Density; integer                                 |
| $\tau$      | Time; shearing stress; torque; sampling interval |
| $\phi$      | Floc volume fraction                             |
| $\psi$      | Flux                                             |
| $\omega$    | Angular velocity                                 |



APPENDIX B

RESULTS OF FLOCCULATION TESTS

Single Compartment; Stake and Stator Paddle

Single Compartment; Turbine Paddle

Multicompartment Systems; Stake and Stator Paddle

## SINGLE COMPARTMENT; STAKE AND STATOR PADDLE

| Run No. | G<br>sec <sup>-1</sup> | T<br>min | Values of $\frac{n^0}{n^1}$ |      |      |         |
|---------|------------------------|----------|-----------------------------|------|------|---------|
|         |                        |          | (1)                         | (2)  | (3)  | Average |
| 1       | 30                     | 8        | 2.03                        | 1.77 | 2.13 | 1.98    |
| 2       | 45                     | 8        | 2.11                        | 2.10 |      | 2.10    |
| 3       | 60                     | 8        | 2.32                        | 2.19 |      | 2.26    |
| 4       | 90                     | 8        | 2.57                        |      |      | 2.57    |
| 5       | 120                    | 8        | 2.66                        | 2.39 |      | 2.52    |
| 6       | 180                    | 8        | 2.11                        | 2.27 | 2.36 | 2.25    |
| 7       | 240                    | 8        | 1.73                        | 1.89 | 1.78 | 1.80    |
| 8       | 30                     | 12       | 2.12                        |      |      | 2.12    |
| 9       | 45                     | 12       | 2.37                        |      |      | 2.37    |
| 10      | 60                     | 12       | 2.65                        |      |      | 2.65    |
| 11      | 90                     | 12       | 2.58                        |      |      | 2.58    |
| 12      | 120                    | 12       | 2.27                        |      |      | 2.27    |
| 13      | 180                    | 12       | 2.06                        |      |      | 2.06    |
| 14      | 240                    | 12       | 1.69                        |      |      | 1.69    |
| 15      | 30                     | 16       | 2.34                        | 2.40 |      | 2.37    |
| 16      | 45                     | 16       | 2.48                        | 2.56 |      | 2.52    |
| 17      | 60                     | 16       | 2.48                        |      |      | 2.48    |
| 18      | 90                     | 16       | 3.24                        |      |      | 3.24    |
| 19      | 120                    | 16       | 2.75                        |      |      | 2.75    |
| 20      | 180                    | 16       | 2.54                        |      |      | 2.54    |
| 21      | 240                    | 16       | 2.06                        |      |      | 2.06    |
| 22      | 30                     | 24       | 1.95                        | 1.46 | 1.85 | 1.75    |
| 23      | 45                     | 24       | 1.98                        | 1.71 |      | 1.84    |
| 24      | 60                     | 24       | 2.07                        | 1.93 | 1.99 | 2.00    |
| 25      | 90                     | 24       | 2.76                        | 3.74 | 3.11 | 3.06    |
| 26      | 120                    | 24       | 2.82                        | 2.86 |      | 2.84    |
| 27      | 180                    | 24       | 2.25                        | 2.13 | 2.01 | 2.13    |
| 28      | 240                    | 24       | 2.04                        |      |      | 2.04    |

## SINGLE COMPARTMENT; TURBINE PADDLE

| Run No. | G<br>sec <sup>-1</sup> | T<br>min | Values of $\frac{n^0}{\frac{1}{n^1}}$ |      |      |         |
|---------|------------------------|----------|---------------------------------------|------|------|---------|
|         |                        |          | (1)                                   | (2)  | (3)  | Average |
| 1       | 30                     | 8        | 1.54                                  | 1.80 | 1.89 | 1.74    |
| 2       | 45                     | 8        | 1.76                                  | 2.05 | 1.92 | 1.91    |
| 3       | 60                     | 8        | 1.88                                  | 2.22 | 2.10 | 2.07    |
| 4       | 90                     | 8        | 2.04                                  | 2.39 | 2.30 | 2.24    |
| 5       | 120                    | 8        | 2.36                                  | 2.36 |      | 2.36    |
| 6       | 180                    | 8        | 1.75                                  | 1.94 |      | 1.84    |
| 7       | 240                    | 8        | 1.57                                  | 1.65 |      | 1.61    |
| 8       | 30                     | 12       | 1.80                                  | 2.14 | 2.05 | 2.00    |
| 9       | 45                     | 12       | 2.17                                  | 2.49 | 2.00 | 2.22    |
| 10      | 60                     | 12       | 1.92                                  | 2.02 | 2.33 | 2.12    |
| 11      | 90                     | 12       | 2.23                                  | 2.41 | 2.49 | 2.38    |
| 12      | 120                    | 12       | 1.89                                  | 2.24 | 2.41 | 2.18    |
| 13      | 180                    | 12       | 1.71                                  | 1.77 |      | 1.74    |
| 14      | 240                    | 12       | 1.46                                  | 1.54 |      | 1.50    |
| 15      | 30                     | 16       | 2.03                                  | 2.05 |      | 2.04    |
| 16      | 45                     | 16       | 2.00                                  | 2.19 | 2.08 | 2.09    |
| 17      | 60                     | 16       | 2.05                                  | 2.56 | 2.35 | 2.29    |
| 18      | 90                     | 16       | 2.22                                  | 2.40 | 2.91 | 2.51    |
| 19      | 120                    | 16       | 2.01                                  | 2.05 |      | 2.03    |
| 20      | 180                    | 16       | 1.79                                  | 1.79 |      | 1.79    |
| 21      | 240                    | 16       | 1.45                                  | 1.60 |      | 1.52    |
| 22      | 30                     | 24       | 1.94                                  | 2.35 | 2.77 | 2.35    |
| 23      | 45                     | 24       | 2.28                                  | 2.38 | 2.96 | 2.54    |
| 24      | 60                     | 24       | 2.17                                  | 2.58 | 3.42 | 2.72    |
| 25      | 90                     | 24       | 2.31                                  | 2.15 | 3.10 | 2.52    |
| 26      | 120                    | 24       | 2.15                                  | 2.09 |      | 2.12    |
| 27      | 180                    | 24       | 2.13                                  | 2.01 |      | 2.07    |
| 28      | 240                    | 24       | 1.38                                  | 1.56 |      | 1.47    |

## MULTICOMPARTMENT SYSTEMS; STAKE AND STATOR PADDLE

| G<br>sec <sup>-1</sup> | T<br>min | m = 2                         |      |      |         | m = 4                         |      |      |         |
|------------------------|----------|-------------------------------|------|------|---------|-------------------------------|------|------|---------|
|                        |          | Values of $\frac{n^0}{n_1^m}$ |      |      |         | Values of $\frac{n^0}{n_1^m}$ |      |      |         |
|                        |          | (1)                           | (2)  | (3)  | Average | (1)                           | (2)  | (3)  | Average |
| 30                     | 4        | 1.94                          | 1.92 |      | 1.93    | 3.59                          | 2.78 |      | 3.18    |
| 45                     | 4        | 2.34                          | 2.24 |      | 2.29    | 4.62                          | 3.24 | 5.11 | 4.32    |
| 60                     | 4        | 2.05                          | 2.32 |      | 2.18    | 3.61                          | 3.97 |      | 3.80    |
| 90                     | 4        | 2.48                          | 2.61 |      | 2.55    | 5.94                          | 4.61 |      | 5.28    |
| 120                    | 4        | 2.78                          | 2.90 |      | 2.84    | 4.68                          | 5.27 |      | 4.98    |
| 180                    | 4        | 2.44                          |      |      | 2.44    | 4.08                          | 3.22 |      | 3.65    |
| 240                    | 4        | 1.70                          | 2.65 |      | 2.00    | 3.37                          | 2.22 | 1.88 | 2.16    |
| 30                     | 6        | 2.92                          | 2.13 | 2.85 | 2.63    | 5.00                          | 3.29 | 4.39 | 4.23    |
| 45                     | 6        | 3.23                          | 3.53 |      | 3.38    | 5.53                          | 5.20 |      | 5.36    |
| 60                     | 6        | 3.62                          | 3.36 |      | 3.50    | 6.90                          | 5.06 |      | 5.98    |
| 90                     | 6        | 3.89                          | 3.87 | 3.89 | 3.89    | 6.40                          | 4.92 | 4.56 | 5.30    |
| 120                    | 6        | 3.03                          | 3.63 | 3.89 | 3.52    | 3.87                          |      |      | 3.87    |
| 180                    | 6        | 3.56                          | 2.42 | 3.11 | 3.03    | 3.56                          | 1.94 | 2.95 | 2.82    |
| 240                    | 6        | 2.28                          | 2.44 | 2.46 | 2.40    | 2.38                          | 2.50 | 1.85 | 2.24    |
| 30                     | 8        | 3.45                          | 3.03 | 3.38 | 3.29    | 5.15                          | 3.92 |      | 4.53    |
| 45                     | 8        | 3.65                          | 2.60 |      | 3.12    | 5.31                          | 4.84 |      | 5.08    |
| 60                     | 8        | 3.34                          | 2.95 |      | 3.15    | 6.09                          | 5.46 |      | 5.78    |
| 90                     | 8        | 3.48                          | 3.00 |      | 3.24    | 5.41                          | 5.08 |      | 5.25    |
| 120                    | 8        | 4.10                          | 3.45 |      | 3.78    | 4.63                          | 4.00 |      | 4.31    |
| 180                    | 8        | 2.42                          | 3.03 |      | 2.72    | 2.53                          | 2.86 |      | 2.70    |
| 240                    | 8        | 2.27                          | 2.49 |      | 2.38    | 2.09                          | 2.29 |      | 2.19    |
| 30                     | 12       | 3.57                          | 3.02 | 3.35 | 3.31    | 6.16                          | 4.65 | 7.20 | 6.00    |
| 45                     | 12       | 3.22                          | 3.30 |      | 3.26    | 4.06                          | 4.38 |      | 4.22    |
| 60                     | 12       | 3.46                          | 4.03 |      | 3.75    | 5.10                          | 5.69 |      | 5.40    |
| 90                     | 12       | 2.74                          | 3.63 |      | 3.20    | 3.59                          | 4.14 |      | 3.86    |
| 120                    | 12       | 3.27                          | 3.30 |      | 3.28    | 4.43                          | 3.87 |      | 4.15    |
| 180                    | 12       | 1.82                          | 1.64 |      | 1.73    | 1.71                          | 2.04 |      | 1.88    |
| 240                    | 12       | 2.12                          | 1.76 |      | 1.94    | 1.87                          | 1.66 |      | 1.76    |

APPENDIX C

STATISTICAL ANALYSIS OF FLOCCULATION TESTS RESULTS

## STATISTICAL ANALYSIS OF FLOCCULATION TESTS RESULTS

The standard deviation of the individual observations within a single sample could not be calculated in the regular way due to the small size of the samples. Hence, it was assumed that all samples had the same coefficient of variance calculated as follows:

$$C_V = \frac{1}{\sum N_j - 1} \sum_j \sum_i \left( s_{ij} - \bar{X}_j \right)^2 / \bar{X}_j^2 \quad (129)$$

Where  $C_V$  is the coefficient of variance;  $X_{ij}$  is the value of  $n_i^0/n_i^1$  belonging to a sample with average  $\bar{X}_j$ .  $\sum N_j$  is the total number of points used in the calculation shown in Table VII.

The standard deviation for any sample with average  $\bar{X}$  was then calculated by

$$S = C_V \bar{X} \quad (130)$$

The detailed calculation of P as given in Table I in the body of the report are presented in Table VIII.



TABLE VII  
CALCULATION OF THE COEFFICIENT OF VARIANCE

| J       | G<br>sec <sup>-1</sup> | T<br>min | Paddle<br>Type    | Values of $X_{1j}$ |      |      |      | $\bar{X}_1$ | $10^4 (X_{1j} - \bar{X}_1)^2 / \bar{X}_1^2$ |       |       |      | $N_j$  | $10^4 \Sigma (X_{1j} - \bar{X}_1)^2 / \bar{X}_1^2$ |
|---------|------------------------|----------|-------------------|--------------------|------|------|------|-------------|---------------------------------------------|-------|-------|------|--------|----------------------------------------------------|
|         |                        |          |                   | 1=1                | 1=2  | 1=3  | 1=4  |             | 1=1                                         | 1=2   | 1=3   | 1=4  |        |                                                    |
| 1       | 30                     | 8        | Stator and Stator | 2.03               | 1.77 | 2.13 |      | 1.98        | 6.4                                         | 112.5 | 51.6  |      | 3      | 170.5                                              |
| 2       | 45                     | 8        |                   | 2.11               | 2.10 |      |      | 2.10        |                                             | 0     |       |      | 2      | .2                                                 |
| 3       | 60                     | 8        |                   | 2.32               | 2.19 |      |      | 2.26        | 7.3                                         | 9.6   |       |      | 2      | 16.6                                               |
| 4       | 30                     | 8        |                   | 2.57               |      |      |      | 2.57        |                                             |       |       |      |        |                                                    |
| 5       | 120                    | 8        |                   | 2.66               | 2.39 |      |      | 2.52        | 30.9                                        | 26.6  |       |      | 2      | 57.5                                               |
| 6       | 180                    | 8        |                   | 2.11               | 2.27 | 2.36 |      | 2.25        | 36.8                                        | .8    | 24.9  |      | 3      | 64.5                                               |
| 7       | 240                    | 8        |                   | 1.73               | 1.89 | 1.78 |      | 1.80        | 15.1                                        | 25.0  | 1.2   |      | 3      | 41.3                                               |
| 8       | 50                     | 12       |                   | 2.12               |      |      |      | 2.12        |                                             |       |       |      |        |                                                    |
| 9       | 45                     | 12       |                   | 2.37               |      |      |      | 2.37        |                                             |       |       |      |        |                                                    |
| 10      | 60                     | 12       |                   | 2.65               |      |      |      | 2.65        |                                             |       |       |      |        |                                                    |
| 11      | 90                     | 12       |                   | 2.98               |      |      |      | 2.98        |                                             |       |       |      |        |                                                    |
| 12      | 120                    | 12       |                   | 2.27               |      |      |      | 2.27        |                                             |       |       |      |        |                                                    |
| 13      | 180                    | 12       |                   | 2.06               |      |      |      | 2.06        |                                             |       |       |      |        |                                                    |
| 14      | 240                    | 12       |                   | 1.69               |      |      |      | 1.69        |                                             |       |       |      |        |                                                    |
| 15      | 30                     | 16       |                   | 2.34               | 2.10 |      |      | 2.37        | 1.6                                         | 1.6   |       |      | 2      | 5.2                                                |
| 16      | 45                     | 16       |                   | 2.48               | 2.56 |      |      | 2.52        | 2.5                                         | 2.5   |       |      | 2      | 5.0                                                |
| 17      | 60                     | 16       |                   | 2.48               |      |      |      | 2.48        |                                             |       |       |      |        |                                                    |
| 18      | 70                     | 16       |                   | 3.14               |      |      |      | 3.14        |                                             |       |       |      |        |                                                    |
| 19      | 120                    | 16       |                   | 2.75               |      |      |      | 2.75        |                                             |       |       |      |        |                                                    |
| 20      | 180                    | 16       |                   | 2.54               |      |      |      | 2.54        |                                             |       |       |      |        |                                                    |
| 21      | 240                    | 16       |                   | 2.06               |      |      |      | 2.06        |                                             |       |       |      |        |                                                    |
| 22      | 50                     | 24       |                   | 2.95               | 2.46 | 2.85 |      | 2.75        | 52.9                                        | 111.1 | 15.2  |      | 3      | 177.2                                              |
| 23      | 45                     | 24       |                   | 2.98               | 2.71 |      |      | 2.84        | 24.5                                        | 20.9  |       |      | 2      | 45.2                                               |
| 24      | 60                     | 24       |                   | 3.07               | 2.93 | 2.99 |      | 3.00        | 5.4                                         | 5.4   | .1    |      | 3      | 10.9                                               |
| 25      | 90                     | 24       |                   | 2.63               | 3.74 | 3.11 | 2.76 | 3.06        | 116.2                                       | 109.2 | 2.7   | 96.1 | 4      | 324.2                                              |
| 26      | 120                    | 24       |                   | 2.82               | 2.83 |      |      | 2.84        | .5                                          | .5    |       |      | 2      | 1.0                                                |
| 27      | 180                    | 24       |                   | 2.25               | 2.13 | ..01 |      | 2.13        | 31.7                                        | 0     | 31.7  |      | 3      | 63.4                                               |
| 28      | 240                    | 24       |                   | 2.04               |      |      |      | 2.04        |                                             |       |       |      |        |                                                    |
| 1       | 30                     | 8        | Turbine           | 1.54               | 1.80 | 1.89 |      | 1.74        | 132.0                                       | 11.9  | 74.5  |      | 3      | 218.2                                              |
| 2       | 45                     | 8        |                   | 1.76               | 2.05 | 1.92 |      | 1.91        | 61.6                                        | 53.7  | .5    |      | 3      | 15.6                                               |
| 3       | 60                     | 8        |                   | 1.98               | 2.22 | 2.10 |      | 2.07        | 84.5                                        | 52.6  | 2.1   |      | 3      | 139.0                                              |
| 4       | 90                     | 8        |                   | 2.04               | 2.39 | 2.30 |      | 2.24        | 79.8                                        | 44.9  | 7.2   |      | 3      | 131.9                                              |
| 5       | 120                    | 8        |                   | 2.36               | 2.36 |      |      | 2.36        | 0                                           | 0     |       |      | 2      | 0.0                                                |
| 6       | 180                    | 8        |                   | 1.75               | 1.94 |      |      | 1.84        | 23.9                                        | 29.5  |       |      | 2      | 58.4                                               |
| 7       | 240                    | 8        |                   | 1.57               | 1.65 |      |      | 1.61        | 9.7                                         | 6.2   |       |      | 2      | 15.9                                               |
| 8       | 30                     | 12       |                   | 1.80               | 2.14 | 2.05 |      | 2.00        | 100.0                                       | 49.7  | 8.5   |      | 3      | 157.3                                              |
| 9       | 45                     | 12       |                   | 2.17               | 2.49 | 2.30 |      | 2.22        | 5.1                                         | 148.0 | 98.4  |      | 3      | 251.5                                              |
| 10      | 60                     | 12       |                   | 1.92               | 2.02 | 2.33 |      | 2.12        | 36.9                                        | 22.2  | 96.0  |      | 3      | 209.1                                              |
| 11      | 90                     | 12       |                   | 2.23               | 2.41 | 2.49 |      | 2.58        | 39.7                                        | 1.6   | 21.5  |      | 3      | 62.6                                               |
| 12      | 120                    | 12       |                   | 1.89               | 2.24 | 2.41 |      | 2.18        | 177.0                                       | 7.6   | 111.5 |      | 3      | 296.1                                              |
| 13      | 180                    | 12       |                   | 1.71               | 1.77 |      |      | 1.74        | 3.0                                         | 3.0   |       |      | 2      | 6.0                                                |
| 14      | 240                    | 12       |                   | 1.46               | 1.74 |      |      | 1.50        | 7.1                                         | 7.1   |       |      | 2      | 14.2                                               |
| 15      | 30                     | 16       |                   | 2.05               | 1.78 | 2.05 | 1.75 | 1.90        | 46.8                                        | 39.9  | 62.5  | 62.5 | 4      | 211.3                                              |
| 16      | 45                     | 16       |                   | 2.00               | 2.19 | 2.38 |      | 2.09        | 18.5                                        | 22.9  | .2    |      | 3      | 41.6                                               |
| 17      | 60                     | 16       |                   | 2.05               | 2.56 | 2.35 |      | 2.29        | 110.0                                       | 139.0 | 6.9   |      | 3      | 235.9                                              |
| 18      | 90                     | 16       |                   | 2.22               | 2.40 | 2.91 |      | 2.51        | 135.0                                       | 19.2  | 254.0 |      | 3      | 400.2                                              |
| 19      | 120                    | 16       |                   | 2.01               | 2.05 |      |      | 2.03        | 1.0                                         | 1.0   |       |      | 2      | 2.0                                                |
| 20      | 180                    | 16       |                   | 1.79               | 1.79 |      |      | 1.79        | 0                                           | 0     |       |      | 2      | 0.0                                                |
| 21      | 240                    | 16       |                   | 1.45               | 1.60 |      |      | 1.52        | 21.2                                        | 27.7  |       |      | 2      | 48.9                                               |
| 22      | 30                     | 24       |                   | 1.94               | 2.55 | 2.77 |      | 2.35        | 304.0                                       | 0     | 319.0 |      | 3      | 625.0                                              |
| 23      | 45                     | 24       |                   | 2.28               | 2.58 | 2.96 |      | 2.54        | 105.0                                       | 39.7  | 27.4  |      | 3      | 172.1                                              |
| 24      | 60                     | 24       |                   | 2.17               | 2.98 | 3.42 |      | 2.72        | 274.0                                       | 26.5  | 662.0 |      | 3      | 962.5                                              |
| 25      | 90                     | 24       |                   | 2.51               | 2.15 | 3.10 |      | 2.52        | 69.5                                        | 216.0 | 550.0 |      | 3      | 815.5                                              |
| 26      | 120                    | 24       |                   | 2.15               | 2.09 |      |      | 2.12        | 2.0                                         | 2.0   |       |      | 2      | 4.0                                                |
| 27      | 180                    | 24       |                   | 2.15               | 2.01 |      |      | 2.08        | 8.4                                         | 8.4   |       |      | 2      | 16.8                                               |
| 28      | 240                    | 24       |                   | 1.58               | 1.56 |      |      | 1.47        | 37.9                                        | 37.9  |       |      | 2      | 75.8                                               |
| Σ = 110 |                        |          |                   |                    |      |      |      |             |                                             |       |       |      | 6187.1 |                                                    |

$$C_V = \frac{1}{\sqrt{109}} \sqrt{0.61871} = 0.075$$

TABLE VIII

STATISTICAL SIGNIFICANCE OF DIFFERENCE IN PERFORMANCE WITH  
DIFFERENT PADDLES, DETAILED CALCULATION

| Run No. | G<br>sec <sup>-1</sup> | T<br>min | $\bar{X}_s$ | $\bar{X}_T$ | N <sub>s</sub> | N <sub>T</sub> | S <sub>s</sub> <sup>2</sup> | S <sub>T</sub> <sup>2</sup> | S <sub>d</sub> | $\bar{X}_d$ | $\frac{\bar{X}_d}{S_d}$ | P<br>% |
|---------|------------------------|----------|-------------|-------------|----------------|----------------|-----------------------------|-----------------------------|----------------|-------------|-------------------------|--------|
| 1       | 30                     | 8        | 1.98        | 1.74        | 3              | 3              | 0.022                       | 0.017                       | 0.114          | 0.24        | 2.11                    | 1.74   |
| 2       | 45                     | 8        | 2.10        | 1.91        | 2              | 3              | 0.025                       | 0.020                       | 0.138          | 0.19        | 1.38                    | 8.38   |
| 3       | 60                     | 8        | 2.26        | 2.07        | 3              | 3              | 0.029                       | 0.024                       | 0.150          | 0.19        | 1.27                    | 10.20  |
| 4       | 90                     | 8        | 2.57        | 2.24        | 1              | 3              | 0.037                       | 0.028                       | 0.215          | 0.33        | 1.54                    | 6.18   |
| 5       | 120                    | 8        | 2.52        | 2.36        | 2              | 2              | 0.036                       | 0.031                       | 0.183          | 0.16        | 0.87                    | 19.22  |
| 6       | 180                    | 8        | 2.25        | 1.84        | 3              | 2              | 0.029                       | 0.019                       | 0.137          | 0.41        | 3.00                    | 0.13   |
| 7       | 240                    | 8        | 1.80        | 1.61        | 3              | 2              | 0.018                       | 0.015                       | 0.115          | 0.19        | 1.64                    | 5.05   |
| 8       | 30                     | 12       | 2.12        | 2.00        | 1              | 3              | 0.025                       | 0.022                       | 0.180          | 0.12        | 0.67                    | 25.14  |
| 9       | 45                     | 12       | 2.37        | 2.22        | 1              | 3              | 0.032                       | 0.028                       | 0.146          | 0.15        | 1.03                    | 15.15  |
| 10      | 60                     | 12       | 2.65        | 2.12        | 1              | 3              | 0.039                       | 0.025                       | 0.217          | 0.53        | 2.45                    | 0.71   |
| 11      | 90                     | 12       | 2.58        | 2.38        | 1              | 3              | 0.037                       | 0.032                       | 0.218          | 0.20        | 0.92                    | 17.88  |
| 12      | 120                    | 12       | 2.27        | 2.18        | 1              | 3              | 0.029                       | 0.027                       | 0.192          | 0.09        | 0.47                    | 31.92  |
| 13      | 180                    | 12       | 2.06        | 1.74        | 1              | 2              | 0.024                       | 0.017                       | 0.180          | 0.32        | 1.78                    | 3.75   |
| 14      | 240                    | 12       | 1.69        | 1.50        | 1              | 2              | 0.016                       | 0.013                       | 0.150          | 0.19        | 1.27                    | 10.20  |
| 15      | 30                     | 16       | 2.37        | 1.90        | 2              | 4              | 0.032                       | 0.020                       | 0.145          | 0.47        | 3.24                    | 0.06   |
| 16      | 45                     | 16       | 2.52        | 2.09        | 2              | 3              | 0.036                       | 0.025                       | 0.162          | 0.43        | 2.66                    | 0.39   |
| 17      | 60                     | 16       | 2.48        | 2.29        | 1              | 3              | 0.035                       | 0.029                       | 0.211          | 0.19        | 0.90                    | 18.41  |
| 18      | 90                     | 16       | 3.14        | 2.51        | 1              | 3              | 0.055                       | 0.035                       | 0.253          | 0.63        | 2.44                    | 2.73   |
| 19      | 120                    | 16       | 2.75        | 2.03        | 1              | 2              | 0.042                       | 0.023                       | 0.231          | 0.72        | 3.12                    | 0.03   |
| 20      | 180                    | 16       | 2.54        | 1.79        | 1              | 2              | 0.036                       | 0.032                       | 0.228          | 0.75        | 3.29                    | 0.05   |
| 21      | 240                    | 16       | 2.06        | 1.52        | 1              | 2              | 0.024                       | 0.013                       | 0.175          | 0.54        | 3.09                    | 0.10   |
| 22      | 30                     | 24       | 2.75        | 2.35        | 3              | 3              | 0.043                       | 0.031                       | 0.157          | 0.40        | 2.55                    | 0.54   |
| 23      | 45                     | 24       | 2.84        | 2.54        | 2              | 3              | 0.045                       | 0.036                       | 0.185          | 0.30        | 1.61                    | 5.37   |
| 24      | 60                     | 24       | 3.00        | 2.72        | 3              | 3              | 0.051                       | 0.041                       | 0.175          | 0.28        | 1.60                    | 5.48   |
| 25      | 90                     | 24       | 3.06        | 2.52        | 4              | 3              | 0.053                       | 0.036                       | 0.158          | 0.54        | 3.42                    | 0.03   |
| 26      | 120                    | 24       | 2.84        | 2.12        | 2              | 2              | 0.045                       | 0.025                       | 0.187          | 0.72        | 3.85                    | 0.01   |
| 27      | 180                    | 24       | 2.13        | 2.07        | 3              | 2              | 0.025                       | 0.024                       | 0.143          | 0.06        | 0.42                    | 33.72  |
| 28      | 240                    | 24       | 2.04        | 1.47        | 1              | 2              | 0.023                       | 0.012                       | 0.170          | 0.57        | 3.35                    | 0.04   |

APPENDIX D  
RESULTS OF TURBULENCE MEASUREMENTS

## RESULTS OF TURBULENCE MEASUREMENTS

The experimental results of turbulence measurements are summarized in Tables IX (for the stake and stator paddle), and X (for the turbine paddle). All experiments were run with water temperature of 21.5°C and with sensor's cold resistance of 6.21 to 6.22 ohms. The operating resistance was set at 6.84 to 6.85 ohms so as to keep an overheat ratio of 1.1 which corresponds to a sensor temperature of approximately 64°C. The standby voltage of the Heat Flux System bridge was kept at 6.80 volts. The same operation conditions were used for calibration of the instrument.

TABLE IX  
TURBULENCE DATA WITH STAKE AND STATOR PADDLE

| Location (1) | G<br>sec <sup>-1</sup> | $\bar{E}_b$<br>Volt | $\bar{e}^2$<br>Volt | $\bar{U}_{eff}$<br>cm/sec | $\frac{\bar{u}^2}{\bar{U}}$<br>% | $\bar{U}$<br>cm/sec | $\frac{\bar{u}^2}{cm^2/sec^2}$ |
|--------------|------------------------|---------------------|---------------------|---------------------------|----------------------------------|---------------------|--------------------------------|
| A<br>Low     | 30                     | 7.8<br>7.9          | 0.15<br>0.15        | 5.0                       | 16.1                             | 4.9                 | 0.48                           |
|              | 120                    | 9.1<br>9.0          | 0.20<br>0.20        | 11.2                      | 12.5                             | 11.0                | 1.89                           |
|              | 240                    | 9.6<br>9.6          | 0.30<br>0.30        | 15.0                      | 17.5                             | 14.5                | 6.45                           |
| A<br>Med.    | 30                     | 8.1<br>8.0          | 0.25<br>0.25        | 5.9                       | 20.4                             | 5.7                 | 1.55                           |
|              | 120                    | 9.1<br>9.4          | 0.30<br>0.35        | 12.4                      | 13.3                             | 12.2                | 2.64                           |
|              | 240                    | 9.7<br>10.2         | 0.40<br>0.40        | 20.0                      | 12.5                             | 19.7                | 6.08                           |
| A<br>High    | 30                     | 7.5                 | 0.25                | 5.0                       | 29.1                             | 2.8                 | 0.67                           |
|              | 120                    | 8.7<br>8.7          | 0.45<br>0.50        | 9.0                       | 31.2                             | 8.3                 | 6.7                            |
|              | 240                    | 9.2<br>9.2          | 0.60<br>0.50        | 11.5                      | 35.0                             | 10.3                | 13.0                           |
| B<br>Low     | 30                     | 7.8<br>7.8          | 0.20<br>0.20        | 4.3                       | 20.4                             | 4.1                 | 0.70                           |
|              | 120                    | 8.9<br>9.0          | 0.40<br>0.40        | 10.3                      | 25.8                             | 9.7                 | 6.25                           |
|              | 240                    | 9.6<br>9.6          | 0.45<br>0.40        | 15.5                      | 25.7                             | 14.7                | 12.2                           |
| B<br>Med.    | 30                     | 8.1<br>4.1          | 0.35<br>0.40        | 5.3                       | 35.3                             | 4.7                 | 2.35                           |
|              | 120                    | 9.1<br>9.2          | 0.55<br>0.60        | 12.0                      | 32.0                             | 10.9                | 12.2                           |
|              | 240                    | 9.9<br>10.0         | 0.60<br>0.70        | 18.8                      | 31.2                             | 17.1                | 28.5                           |
| B<br>High    | 30                     | 7.5<br>7.5          | 0.30<br>0.30        | 3.2                       | 29.8                             | 2.9                 | 0.75                           |
|              | 120                    | 8.8<br>8.9          | 0.55<br>0.55        | 9.5                       | 37.8                             | 8.5                 | 10.5                           |
|              | 240                    | 9.5<br>9.5          | 0.65<br>0.60        | 14.7                      | 34.1                             | 13.2                | 20.5                           |
| C<br>Low     | 30                     | 7.3<br>7.3          | 0.15<br>0.10        | 2.7                       | 17.0                             | 2.6                 | 0.20                           |
|              | 120                    | 8.6<br>8.4          | 0.30<br>0.25        | 7.8                       | 19.5                             | 7.5                 | 2.14                           |
|              | 240                    | 9.2<br>9.2          | 0.30<br>0.35        | 12.2                      | 16.2                             | 11.8                | 5.67                           |
| C<br>Med.    | 30                     | 7.1<br>7.2          | 0.10<br>0.15        | 2.2                       | 18.7                             | 2.1                 | 0.16                           |
|              | 120                    | 8.3<br>8.5          | 0.25<br>0.30        | 6.8                       | 22.4                             | 6.5                 | 2.12                           |
|              | 240                    | 9.0<br>9.0          | 0.35<br>0.40        | 10.8                      | 23.3                             | 10.3                | 3.6                            |
| C<br>High    | 30                     | 7.5<br>7.2          | 0.20<br>0.20        | 2.8                       | 26.6                             | 2.6                 | 0.48                           |
|              | 120                    | 8.6<br>8.6          | 0.40<br>0.40        | 8.2                       | 32.3                             | 7.5                 | 5.9                            |
|              | 240                    | 9.3<br>9.3          | 0.45<br>0.45        | 15.0                      | 29.8                             | 12.2                | 9.9                            |

(1) Locations A, B, and C are as shown in Figure 53. Low, Med., and High refer to 5-in. below mid-depth, mid-depth, and 5-in. above mid-depth, respectively.

TABLE X  
TURBULENCE DATA WITH TURBINE PADDLE

| Location  | G<br>sec <sup>-1</sup> | $\bar{E}_b$<br>Volt | $\bar{e}^2$<br>Volt | $\bar{U}_{eff}$<br>cm/sec | $\frac{\bar{u}^2}{U}$<br>% | $\bar{U}$<br>cm/sec | $\frac{\bar{u}^2}{cm^2/sec^2}$ |
|-----------|------------------------|---------------------|---------------------|---------------------------|----------------------------|---------------------|--------------------------------|
| A<br>Low  | 30                     | 7.8                 | 0.10                | 5.0                       | 5.8                        | 5.0                 | 0.08                           |
|           | 120                    | 8.1                 | 0.15                | 5.5                       | 12.9                       | 5.4                 | 0.49                           |
|           | 240                    | 9.3                 | 0.18                | 11.5                      | 11.2                       | 11.4                | 1.63                           |
| A<br>Med. | 30                     | 6.6                 | 0.10                | 1.5                       | 16.6                       | 1.4                 | 0.05                           |
|           | 120                    | 7.9                 | 0.15                | 5.0                       | 13.3                       | 4.9                 | 0.42                           |
|           | 240                    | 8.5                 | 0.21                | 7.5                       | 15.4                       | 7.3                 | 1.26                           |
| B<br>Low  | 30                     | 7.5<br>7.4          | 0.10<br>0.15        | 3.5                       | 11.5                       | 3.4                 | 0.05                           |
|           | 120                    | 8.9<br>8.9          | 0.20<br>0.25        | 10.0                      | 15.1                       | 9.8                 | 2.19                           |
|           | 240                    | 9.5<br>9.5          | 0.30<br>0.30        | 14.3                      | 17.1                       | 13.9                | 5.65                           |
| B<br>Med. | 30                     | 7.3<br>7.4          | 0.10<br>0.10        | 3.3                       | 10.8                       | 3.2                 | 0.12                           |
|           | 120                    | 8.6<br>8.5          | 0.25<br>0.30        | 7.6                       | 20.4                       | 7.3                 | 2.22                           |
|           | 240                    | 9.5<br>9.4          | 0.35<br>0.30        | 14.1                      | 18.7                       | 13.7                | 6.55                           |
| B<br>High | 30                     | 7.4<br>7.6          | 0.15<br>0.15        | 3.6                       | 15.4                       | 3.5                 | 0.29                           |
|           | 120                    | 8.8<br>8.6          | 0.25<br>0.30        | 7.7                       | 20.4                       | 7.4                 | 2.28                           |
|           | 240                    | 9.4<br>9.7          | 0.40<br>0.40        | 14.2                      | 22.7                       | 13.5                | 9.38                           |
| C<br>Low  | 30                     | 7.1<br>6.8          | 0.10<br>0.10        | 2.2                       | 4.5                        | 2.2                 | 0.02                           |
|           | 120                    | 8.2<br>8.3          | 0.25<br>0.30        | 6.4                       | 21.5                       | 6.1                 | 1.72                           |
|           | 240                    | 9.0<br>8.8          | 0.35<br>0.40        | 10.0                      | 25.3                       | 9.4                 | 5.65                           |
| C<br>Med. | 30                     | 7.0<br>7.0          | 0.15<br>0.15        | 2.4                       | 17.4                       | 2.3                 | 0.16                           |
|           | 120                    | 8.0<br>8.2          | 0.35<br>0.30        | 5.5                       | 26.1                       | 5.2                 | 1.85                           |
|           | 240                    | 8.7<br>8.8          | 0.50<br>0.50        | 9.0                       | 34.3                       | 8.1                 | 7.72                           |
| C<br>High | 30                     | 6.5<br>7.0          | 0.20<br>0.10        | 2.0                       | 16.2                       | 1.9                 | 0.09                           |
|           | 120                    | 8.0<br>8.1          | 0.40<br>0.30        | 5.4                       | 28.6                       | 5.0                 | 2.05                           |
|           | 240                    | 8.5<br>8.5          | 0.50<br>0.50        | 7.5                       | 37.0                       | 6.7                 | 6.15                           |



APPENDIX E  
CALCULATION OF AVERAGE VALUES OF  $\overline{u^2}$

TABLE XI

AVERAGE  $\overline{u^2}$  FOR  $G = 30 \text{ sec}^{-1}$ , TURBINE PADDLE

| Location   | Elevation | Weight | $\overline{u^2}$ | Weight x $\overline{u^2}$ |
|------------|-----------|--------|------------------|---------------------------|
| A          | Low       | 0.13   | 0.08             | 0.010                     |
| A          | Med.      | 0.13   | 0.42             | 0.055                     |
| B          | Low       | 0.33   | 0.15             | 0.050                     |
| B          | Med.      | 0.33   | 0.12             | 0.040                     |
| B          | High      | 0.33   | 0.29             | 0.096                     |
| C          | Low       | 0.54   | 0.02             | 0.011                     |
| C          | Med.      | 0.54   | 0.16             | 0.086                     |
| C          | High      | 0.54   | 0.09             | 0.049                     |
| $\Sigma =$ |           | 2.87   |                  | 0.397                     |

$$\overline{(\overline{u^2})} = \frac{0.397}{2.87} = 0.138 \text{ cm}^2/\text{sec}^2$$

TABLE XII

AVERAGE  $\overline{u^2}$  FOR  $G = 120 \text{ sec}^{-1}$ , TURBINE PADDLE

| Location   | Elevation | Weight | $\overline{u^2}$ | Weight x $\overline{u^2}$ |
|------------|-----------|--------|------------------|---------------------------|
| A          | Low       | 0.13   | 0.49             | 0.064                     |
| A          | Med.      | 0.13   | 0.42             | 0.055                     |
| B          | Low       | 0.33   | 2.19             | 0.723                     |
| B          | Med.      | 0.33   | 2.22             | 0.733                     |
| B          | High      | 0.33   | 2.28             | 0.753                     |
| C          | Low       | 0.54   | 1.72             | 0.928                     |
| C          | Med.      | 0.54   | 1.72             | 0.928                     |
| C          | High      | 0.54   | 2.05             | 1.108                     |
| $\Sigma =$ |           | 2.87   |                  | 5.364                     |

$$\overline{(\overline{u^2})} = \frac{5.364}{2.87} = 1.87 \text{ cm}^2/\text{sec}^2$$

TABLE XIII

AVERAGE  $\overline{u^2}$  FOR  $G = 240 \text{ sec}^{-1}$ , TURBINE PADDLE

| Location   | Elevation | Weight | $\overline{u^2}$ | Weight x $\overline{u^2}$ |
|------------|-----------|--------|------------------|---------------------------|
| A          | Low       | 0.13   | 1.63             | 0.220                     |
| A          | Med.      | 0.13   | 1.26             | 0.160                     |
| B          | Low       | 0.33   | 5.65             | 1.865                     |
| B          | Med.      | 0.33   | 6.55             | 2.160                     |
| B          | High      | 0.33   | 9.38             | 3.095                     |
| C          | Low       | 0.54   | 5.65             | 3.050                     |
| C          | Med.      | 0.54   | 7.72             | 4.170                     |
| C          | High      | 0.54   | 6.15             | 3.320                     |
| $\Sigma =$ |           | 2.87   |                  | 18.040                    |

$$\overline{(\overline{u^2})} = \frac{18.040}{2.87} = 6.28 \text{ cm}^2/\text{sec}^2$$

TABLE XIV

AVERAGE  $\overline{u^2}$  FOR  $G = 30 \text{ sec}^{-1}$ , STAKE AND STATOR PADDLE

| Location   | Elevation | Weight | $\overline{u^2}$ | Weight x $\overline{u^2}$ |
|------------|-----------|--------|------------------|---------------------------|
| A          | Low       | 0.13   | 0.48             | 0.062                     |
| A          | Med.      | 0.13   | 1.35             | 0.175                     |
| A          | High      | 0.13   | 0.67             | 0.089                     |
| B          | Low       | 0.33   | 0.70             | 0.231                     |
| B          | Med.      | 0.33   | 2.75             | 0.907                     |
| B          | High      | 0.33   | 0.75             | 0.247                     |
| C          | Low       | 0.54   | 0.20             | 0.108                     |
| C          | Med.      | 0.54   | 0.16             | 0.086                     |
| C          | High      | 0.54   | 0.48             | 0.259                     |
| $\Sigma =$ |           | 3.00   |                  | 2.164                     |

$$\overline{(\overline{u^2})} = \frac{2.164}{3.00} = 0.723 \text{ cm}^2/\text{sec}^2$$

TABLE XV

AVERAGE  $\overline{u^2}$  FOR  $G = 120 \text{ sec}^{-1}$ , STAKE AND STATOR PADDLE

| Location | Elevation | Weight     | $\overline{u^2}$ | Weight x $\overline{u^2}$ |
|----------|-----------|------------|------------------|---------------------------|
| A        | Low       | 0.13       | 1.89             | 0.246                     |
| A        | Med.      | 0.13       | 2.64             | 0.343                     |
| A        | High      | 0.13       | 6.70             | 0.870                     |
| B        | Low       | 0.33       | 6.25             | 2.062                     |
| B        | Med.      | 0.33       | 12.20            | 4.025                     |
| B        | High      | 0.33       | 10.30            | 3.400                     |
| C        | Low       | 0.54       | 2.14             | 1.155                     |
| C        | Med.      | 0.54       | 2.12             | 1.145                     |
| C        | High      | 0.54       | 5.9              | 3.185                     |
|          |           | $\Sigma =$ | 3.00             | 16.431                    |

$$\overline{(\overline{u^2})} = \frac{16.431}{3.00} = 5.49 \text{ cm}^2/\text{sec}^2$$

TABLE XVI

AVERAGE  $\overline{u^2}$  FOR  $G = 240 \text{ sec}^{-1}$ , STAKE AND STATOR PADDLE

| Location | Elevation | Weight     | $\overline{u^2}$ | Weight x $\overline{u^2}$ |
|----------|-----------|------------|------------------|---------------------------|
| A        | Low       | 0.13       | 6.45             | 0.840                     |
| A        | Med.      | 0.13       | 6.08             | 0.790                     |
| A        | High      | 0.13       | 13.00            | 1.690                     |
| B        | Low       | 0.33       | 12.20            | 4.030                     |
| B        | Med.      | 0.33       | 28.50            | 9.415                     |
| B        | High      | 0.33       | 20.30            | 6.700                     |
| C        | Low       | 0.54       | 3.67             | 1.982                     |
| C        | Med.      | 0.54       | 5.80             | 3.130                     |
| C        | High      | 0.54       | 9.90             | 5.350                     |
|          |           | $\Sigma =$ | 3.00             | 33.927                    |

$$\overline{(\overline{u^2})} = \frac{33.927}{3.00} = 11.31 \text{ cm}^2/\text{sec}^2$$

APPENDIX F  
COMPUTER PROGRAM FOR SPECTRUM ANALYSIS

```

$JOB      5037      9,200,100, .2      ARGAMAN
$SETUP UT8      2551
$TBJOB TURB
$INFTC SPEC
      DIMENSION PDATA(4500), UDATA(13500), ACOV(1501),
      1TSPEC(500), FR(500), ESPEC(500), ESU(500), USFT(500)
      EQUIVALENCE(PDATA,UDATA)
      INTEGER EOF,ERR
      LOGICAL DEBUG
      ASSIGN 120 TO EOF
      ASSIGN 130 TO ERR
      NZ=0
      L = 0
1000 READ 1, NPTS, DEBUG, V1, V2, V3, V4, V5, NV6, NV7, HOLE, ELEV,
      6POS1, SR, IG, PATYP, INDEX
C MAKE SURE THAT NPTS IS EXACTLY DIVISIBLE BY 3 AND ENDS IN 00
      1 FORMAT(15, L5, 5F5.0, 2I3, 3A3,F6.0, I3, A5, I1)
      L = L+1
      NLAGS = NPTS/NV6
      MPTS = NPTS/3
      NLAGS1 = NLAGS + 1
      CALL READ(MPTS, PDATA, EOF, ERR)
      IF(INDEX)6,6,7
      6 PRINT 3
      PRINT 2, L
      PRINT 8
      8 FORMAT(28H DUMMY OR BAD RECORD SKIPPED)
      GO TO 1000
      7 PRINT 3
      3 FORMAT (11H1RECORD NO.)
      PRINT 2, L
      2 FORMAT (1H , I3)
      PRINT 71
      71 FORMAT(52H HOLE ELEV. POSIT. SAMPL RATE G PADDLE TYPE)
      PRINT 72, HOLE, ELEV, POS1, SR, IG, PATYP
      72 FORMAT(1H0,1X,A3,3X,A3,4X,A3,5X,F8.1,4X,I3,6X,A5)
      IF (DEBUG) PRINT 399, (PDATA(I), I=1,10)
      CALL UNPACK (MPTS, PDATA, UDATA)
      IF (DEBUG) PRINT 400, (UDATA(I), I=1,30)
      399 FORMAT (1H0/(1H 012.4X))
      400 FORMAT (1H0/(1H F12.6,4X))
      DO 511 = 1,NPTS
      IF(2048.0 - UDATA(I))50,50,51
      50 PRINT 52, I
      52 FORMAT(11H0UDATA NO. 15,12H IS TOO BIG)
      GO TO 1000
      51 CONTINUE
      DO 3001 I = 1,NPTS
      3001 UDATA(I)=((((UDATA(I)/1.0235E5-.01)/(V5!+V4)**2-V1)/V2)**V3
      IF (DEBUG) PRINT 400, (UDATA(I), I=1,30)
      IF (DEBUG) PRINT 400, (UDATA(I), I=1,NPTS,300)
      CALL DETRNO(UDATA,NPTS,UDATA,I)
      CALL AJTCOV(UDATA,NPTS,ACOV,NLAGS,NV7)
      PRINT 40
      40 FORMAT (1H1,17X3HLAG16X4HACOV)
      SPEC01
      SPEC02
      SPEC03
      SPEC04
      SPEC05
      SPEC06
      SPEC07
      SPEC08
      SPEC09
      SPEC10
      SPEC11
      SPEC12
      SPEC13
      SPEC14
      SPEC15
      SPEC16
      SPEC17
      SPEC18
      SPEC19
      SPEC20
      SPEC21
      SPEC22
      SPEC23
      SPEC24
      SPEC25
      SPEC26
      SPEC27
      SPEC28
      SPEC29
      SPEC30
      SPEC31
      SPEC32
      SPEC33
      SPEC34
      SPEC35
      SPEC36
      SPEC37
      SPEC38
      SPEC39
      SPEC40
      SPEC41
      SPEC42
      SPEC43
      SPEC44
      SPEC45
      SPEC46
      SPEC47
      SPEC48
      SPEC49
      SPEC50
      SPEC51

```



```

      PRINT 41, NZ, ACOV(1)
      PRINT 41,(1, ACOV(I+1), I = 1,NLAGS)
41  FORMAT (1H 16X14,F20.5)
      PRINT 42
42  FORMAT(1H1,5X4HNFRQBX4HUSFT7X13HFOUT(SMOOTH)3X12HFREQ(C.P.S.)4X12
6HENERG. SPEC.6X20HENERG. SPEC.(SMOOTH))
      CALL FOURTR(ACOV,NLAGS,TSPEC,USFT,3)
      FNV7 = NV7
      FNLG5 = NLAGS
      DO11 I=1,NLAGS1
      FI = 1
      FR(I) = (FI*SR)/(2.0*FNLG5*FNV7)
      ESU(I) = (USFT(I)*FNLG5*FNV7)/(1.5*SR)
11  ESPEC(I) = (TSPEC(I)*FNLG5*FNV7)/(1.5*SR)
      PRINT 43,NZ,USFT(1),TSPEC(1),NZ,ESU(1),ESPEC(1)
      PRINT 43,(1,USFT(I+1),TSPEC(I+1),FR(I),ESU(I+1),ESPEC(I+1),I=1,NLAGS1)
43  FORMAT(1H ,5X,13,4X,F12.7,3X,F12.7,4X,F12.4,2X,F15.9,7X,F15.9)
      GO TO 1000
120 PRINT 121
121 FORMAT (4H1EOF)
      GO TO 1000
130 PRINT 131
131 FORMAT (4H1ERR)
      GO TO 1000
      END
$IBMAP READ 35,MONSYM,LIST
READ SAVE 1
      CLA* 3,4
      ALS 18
      STD COMMD
      CLA 4,4
      STA COMMD
      STA PZE
      CLA* 5,4
      STA EOF
      CLA* 6,4
      STA ERR
      SKA SV4,4
      LAC SYSUT8,4
      ZET 1,4
      TRA *-1
      CLA LCOM
      STO 1,4
      TSX .ACTV,4
      PZE SYSUT8
      LAC SYSUT8,4
      ZET 1,4
      TRA *-1
      LDI 3,4
      LFT 200000
      EOF TRA *-
      ERR LFT 100000
      ERR TRA *-
      CAL 3,4

```

SPEC52  
 SPEC53  
 SPEC54  
 SPEC55  
 SPEC56  
 SPEC57  
 SPEC58  
 SPEC59  
 SPEC60  
 SPEC61  
 SPEC62  
 SPEC63  
 SPEC64  
 SPEC65  
 SPEC66  
 SPEC67  
 SPEC68  
 SPEC69  
 SPEC70  
 SPEC71  
 SPEC72  
 SPEC73  
 SPEC74  
 SPEC75  
 SPEC76  
 READ01  
 READ02  
 READ03  
 READ04  
 READ05  
 READ06  
 READ07  
 READ08  
 READ09  
 READ10  
 READ11  
 READ12  
 READ13  
 READ14  
 READ15  
 READ16  
 READ17  
 READ18  
 READ19  
 READ20  
 READ21  
 READ22  
 READ23  
 READ24  
 READ25  
 READ26  
 READ27  
 READ28

|         |                |                  |          |
|---------|----------------|------------------|----------|
| SV4     | SUB            | PZE              | READ29   |
|         | AXT            | ** , 4           | READ30   |
|         | STA*           | 3 , 4            | READ31   |
|         | RETURN         | READ             | READ32   |
| LCCM    | PZF            | COMMD , 1        | READ33   |
| COMMD   | IORT           | ** , * - *       | READ34   |
| PZE     | PZE            | **               | READ35   |
|         | END            |                  |          |
| \$IDMAP | UNPACK         | 35 , LIST        | UNPACK01 |
|         | ENTRY          | UNPACK           | UNPACK02 |
| UNPACK  | SXA            | UX2 , 2          | UNPACK03 |
|         | SXA            | UX1 , 1          | UNPACK04 |
|         | CLA*           | 3 , 4            | UNPACK05 |
|         | PAX            | 0 , 1            | UNPACK06 |
|         | SXD            | LIM , 1          | UNPACK07 |
|         | ADM            | 4 , 4            | UNPACK08 |
|         | STA            | GET              | UNPACK09 |
|         | CLA*           | 3 , 4            | UNPACK10 |
|         | ALS            | 1                | UNPACK11 |
|         | ADD*           | 3 , 4            | UNPACK12 |
|         | ADM            | 5 , 4            | UNPACK13 |
|         | STA            | PUT1             | UNPACK14 |
|         | ADD            | = 1              | UNPACK15 |
|         | STA            | PUT2             | UNPACK16 |
|         | ADD            | = 1              | UNPACK17 |
|         | STA            | PUT3             | UNPACK18 |
|         | AXT            | 1 , 2            | UNPACK19 |
|         | AXT            | 3 , 1            | UNPACK20 |
| GET     | LDQ            | ** , 2           | UNPACK21 |
|         | ZAC            |                  | UNPACK22 |
|         | LGL            | 12               | UNPACK23 |
| PUT1    | SLW            | ** , 1           | UNPACK24 |
|         | ZAC            |                  | UNPACK25 |
|         | LGL            | 12               | UNPACK26 |
| PUT2    | SLW            | ** , 1           | UNPACK27 |
|         | ZAC            |                  | UNPACK28 |
|         | LGL            | 12               | UNPACK29 |
| PUT3    | SLW            | ** , 1           | UNPACK30 |
| * NOW   | FLOAT THE DATA |                  | UNPACK31 |
|         | CAL*           | PUT1             | UNPACK32 |
|         | ORA            | = 02330000000000 | UNPACK33 |
|         | FAD            | = 02330000000000 | UNPACK34 |
|         | SLW*           | PUT1             | UNPACK35 |
|         | CAL*           | PUT2             | UNPACK36 |
|         | ORA            | = 02330000000000 | UNPACK37 |
|         | FAD            | = 02330000000000 | UNPACK38 |
|         | SLW*           | PUT2             | UNPACK39 |
|         | CAL*           | PUT3             | UNPACK40 |
|         | ORA            | = 02330000000000 | UNPACK41 |
|         | FAD            | = 02330000000000 | UNPACK42 |
|         | SLW*           | PUT3             | UNPACK43 |
|         | TXI            | ** , 1 , 3       | UNPACK44 |
|         | TXI            | ** , 2 , 1       | UNPACK45 |
| LIM     | TXL            | GET , 2 , **     | UNPACK46 |
| UX1     | AXT            | ** , 1           |          |

```

UX2   AXT   **,2
      TRA   1.4
      END
$IBFTC MERE
      SUBROUTINE DETRND(X,N,Y,NDEG)
C      BE DETRND MEAN AND LINEAR TREND REMOVAL
      DIMENSION X(1),Y(1)
      AN=N
      S2=0.0
      S1=0.0
      IF (NDEG) 11,11,20
11     DO 12 I=1,N
12     S1=S1+X(I)
      AVEX=S1/AN
      DO 15 I=1,N
15     Y(I)=X(I)-AVEX
      PRINT 200,AVEX
200    FORMAT(16HREMOVE MEAN OF 1PE15.7)
      RETURN
20     DO 22 I=1,N
      S1=S1+X(I)
22     S2=S2+S1
      AVE1=0.5*(AN+1.0)
      AVEX=S1/AN
      SLOPE=-12.0*(S2-AVE1*S1)/(AN*(AN**2-1.0))
      DO 28 I=1,N
      A1=I
28     Y(I)=X(I)-AVEX-SLOPE*(A1-AVE1)
      PRINT 300,AVEX,SLOPE
300    FORMAT(16HREMOVE MEAN OF 1PE15.7,32H AND LINEAR TREND WITH SLOPE
1OF 1PE15.7)
      RETURN
      END
$IBFTC AUTO
      SUBROUTINE AUTCOV(X,N,Y,L,NV7)
C      BE AUTCOV AUTOCOVARANCE SUBROUTINE
      DIMENSION X(1),Y(1)
      L1=L+1
      AN=N
      SUM=0.0
      DO 11 I=1,N
11     SUM=SUM+X(I)
      E=(SUM/AN)**2
      DO 12 I=1,L1
      Y(I)=0.0
      I1=N-I*N*NV7+NV7
      DO 13 J=1,I1
      J1=J+NV7*(I-1)
13     Y(I)=Y(I)+X(J)*X(J1)
12     Y(I)=Y(I)/FLOAT(I1)-E
      PRINT 200,L
200    FORMAT(20HCAUTOCOVARANCE WITH 14.5H LAGS)
      RETURN
      END
$IBFTC FOUR
      DETRND01
      DETRND02
      DETRND03
      DETRND04
      DETRND05
      DETRND06
      DETRND07
      DETRND08
      DETRND09
      DETRND10
      DETRND11
      DETRND12
      DETRND13
      DETRND14
      DETRND15
      DETRND16
      DETRND17
      DETRND18
      DETRND19
      DETRND20
      DETRND21
      DETRND22
      DETRND23
      DETRND24
      DETRND25
      DETRND26
      DETRND27
      AUTCOV01
      AUTCOV02
      AUTCOV03
      AUTCOV04
      AUTCOV05
      AUTCOV06
      AUTCOV07
      AUTCOV08
      AUTCOV09
      AUTCOV10
      AUTCOV11
      AUTCOV12
      AUTCOV13
      AUTCOV14
      AUTCOV15
      AUTCOV16
      AUTCOV17
      AUTCOV18

```

```

C      SUBROUTINE FOURTR(X,N,Y,Z,IND)
      RE FOURTR    FOURIER TRANSFORM SUBROUTINE
      DIMENSION X(2),Y(2),TAB(3000),Z(2)
      COMMON TAB
      AN=N
      N1=N+1
      N2=N+N
      ANG=3.14159265/AN
      CS=COS(ANG)
      SN=SIN(ANG)
      GOTO(10,11,10,11),IND
C      COSINE TABLE
10     TAB(1)=1.0
      W=0.0
      DO12 I=2,N2
      TAB(I)=TAB(I-1)*CS-W*SN
12     W=W*CS+TAB(I-1)*SN
      ONE=1.0
      U=X(1)
      GOTO13
C      SINE TABLE
11     TAB(1)=0.0
      W=1.0
      DO14 I=2,N2
      TAB(I)=TAB(I-1)*CS+W*SN
14     W=W*CS-TAB(I-1)*SN
      ONE=0.0
      U=0.0
13     DO15 I=1,N1
      I1=I-1
      W=0.0
      DO16 J=2,N
      K=MOD(I1*(J-1),N2)
16     W=W+X(J)*TAB(K+1)
      Y(I)=(U+2.0*W+ONE*X(N+1))/AN
      Z(I)=Y(I)
15     ONE=-ONE
      GOTO(17,18,19,21),IND
17     PRINT200
      GOTO20
18     PRINT201
      GOTO20
19     W2=(Y(1)+Y(2))/2.0
      W3=(Y(N)+Y(N+1))/2.0
      PRINT202
      GOTO22
21     W2=0.0
      W3=0.0
      PRINT203
22     DO23 I=2,N
      W=(Y(I-1)+Y(I+1))/4.0+Y(I)/2.0
      Y(I-1)=W2
23     W2=W
      Y(N)=W2
      Y(N+1)=W3
20     RETURN
200    FORMAT(17H0COSINE TRANSFORM)
201    FORMAT(15HCSINE TRANSFORM)
202    FORMAT(26H0SMOOTHED COSINE TRANSFORM)
203    FORMAT(24H0SMOOTHED SINE TRANSFORM)
      END
$DATA
$EOF

```

```

FOURTR01
FOURTR02
FOURTR03
FOURTR04
FOURTR05
FOURTR06
FOURTR07
FOURTR08
FOURTR09
FOURTR10
FOURTR11
FOURTR12
FOURTR13
FOURTR14
FOURTR15
FOURTR16
FOURTR17
FOURTR18
FOURTR19
FOURTR20
FOURTR21
FOURTR22
FOURTR23
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FOURTR41
FOURTR42
FOURTR43
FOURTR44
FOURTR45
FOURTR46
FOURTR47
FOURTR48
FOURTR49
FOURTR50
FOURTR51
FOURTR52
FOURTR53
FOURTR54
FOURTR55
FOURTR56
FOURTR57
FOURTR58
FOURTR59
FOURTR60

```

APPENDIX G

COMMENTS ON SPECTRUM MEASUREMENTS

## COMMENTS ON SPECTRUM MEASUREMENTS

The computer program used for calculating the correlation function and the energy spectrum is given in Appendix F.

The accuracy of the calculated energy spectrum is a function of the experimental and computational procedures. Hence, several decisions with regard to these procedures had to be made prior to the experiments.

The first question was the desired and attainable frequency response, i.e., the highest frequency that can be detected by the system.

The desired frequency response is the frequency which corresponds to eddies of the order of 100 microns, which is the typical size of flocs and the length scale of the collision process. From Equations 4 and 45, the frequency and scale are related by

$$f = \frac{1}{2\pi} G^{\frac{2}{3}} \nu^{\frac{1}{3}} \lambda^{-\frac{2}{3}} . \quad (131)$$

For the maximum  $G$  of  $240 \text{ sec}^{-1}$ , with  $\nu = 10^{-2} \text{ cm}^2/\text{sec}$  and  $\lambda = 10^{-2} \text{ cm}$ , the frequency is 30 cps. Since the eddies governing the collision process are smaller than 100 microns it was decided that a frequency response of the order of 200 cps is required.

The frequency response of the hot-film anemometer and the associated electronics was determined experimentally and found to be of the order of 2000 cps. This frequency is considerably larger than the required 200 cps.

The frequency response of the computation procedure depends on the speed of the digital recording. If the time interval between two successive data points is  $\tau$ , the frequency response is  $1/2\tau$ . For a frequency response of 200 cps, the required sampling time interval is  $1/400 \text{ sec}$ , corresponding to a sampling rate of 400 samples per second. The sampling rates used in this experiment were either 468.7 or 1875 samples per second which exceed the required rate.

The duration of each run was limited by the memory capacity of the computer. Each record was limited to 13,500 data points corresponding to record length of 7.2 or 28.5 seconds depending on sampling rate.

The accuracy of the computed correlation function depends on the number of time lags used. Preliminary experiments have shown that the correlation function approaches zero for time lags of about 0.5 seconds. Thus the largest time lag was of that order. Since most of the computer



time was spent on calculating the correlation function it was necessary to limit the number of lags to a practical value. The largest number of lags used was 300. Since the lags had to be integral multiples of the sampling interval the correlation function was calculated for time lags of

$$\Delta t = m\tau$$

$$m = 0, 1, \dots M$$

with  $M_{\max} = 300$ .  $\rho$  was selected so that  $M\rho > 0.3$  sec.

The accuracy of the calculated spectrum function is determined by the number of degrees of freedom,  $k$ , in the calculation of the correlation function.  $k$  is given by

$$k \approx \frac{2 T_n'}{\tau_{\max}} \quad (132)$$

where  $T_n'$  is the duration of the record and  $\tau_{\max}$  is the largest time lag used in computation of the correlation function. For a record length of 7.2 sec and a maximum lag of 0.3 sec  $k \approx 50$ . For this number of degrees of freedom there is 80 percent confidence that the true value of the spectrum function  $E$  lies between  $E/0.75$  and  $E/1.26$  where  $E$  is the calculated spectrum function.

APPENDIX H

TESTING PERFORMANCE PARAMETER USING LABORATORY-  
AND FULL-SCALE FLOCCULATORS

# TESTING PERFORMANCE PARAMETER USING LABORATORY- AND FULL-SCALE FLOCCULATORS

In order to test the validity of Equation 85 a series of tests were carried out to compare the laboratory scale flocculation apparatus with a full-scale flocculator. The laboratory apparatus was placed in operation treating the raw water at the Upper San Leandro treatment plant of the East Bay Municipal Utility District. Both the laboratory and plant units were operated with four compartments which previous studies had shown to have essentially identical residence time distribution characteristics. The laboratory unit was operated with the "stake" type paddles, but without stators, and was comparable in geometry to the plant scale units. Over the ten-day period of study the influent turbidity ranged from 6.2 to 8.2 Jackson units and the alum dose varied from 20.5 to 21.4 mg/l. Temperatures ranged from 52 to 53°F. The prototype operated at a flow of 10 mgd with a  $T/4$  of 7.5 minutes and a constant  $G$ . The laboratory unit was operated in an identical manner, except that  $G$  was varied to permit the calculation of  $K_A$  and  $K_B$  and to indicate if the prototype performance could be improved. The  $G$  of the prototype was estimated to be  $20 \text{ sec}^{-1}$ . Samples of flocculated water were removed from the laboratory and prototype flocculators and settled in the established manner. The results were expressed as the ratio of turbidities,  $n_1^0/n_1^m$ , and are shown in Figure 67.

The model and prototype results are nearly the same for  $m=1$  and  $m=3$ , whereas the deviation was appreciable for  $m=2$  and  $m=4$ . It was also evident that the flocculation-sedimentation performance could be improved by increasing the energy input. Additional performance measurements were made with the laboratory flocculator operating at  $T/m = 12 \text{ min}$  and at values of  $G$  up to  $120 \text{ sec}^{-1}$ . The constants at both values of  $T/m$  and for both laboratory and prototype flocculators were

$$K_A = 1.80 \times 10^{-5} \quad \text{and} \quad K_B = 0.80 \times 10^{-7} \text{ sec}$$

Although the limited data presented in this appendix do not provide a conclusive argument for the validity of the pilot plant for predicting prototype flocculator performance, they do support the proposed formula (e.g., Equation 85) and the contention that the flocculation process can be modeled. However, we should be reminded that performance also depends on sedimentation and that the simple batch test used to compare flocculators does not provide an absolute prediction of how the flocculation-sedimentation system will perform in practice.

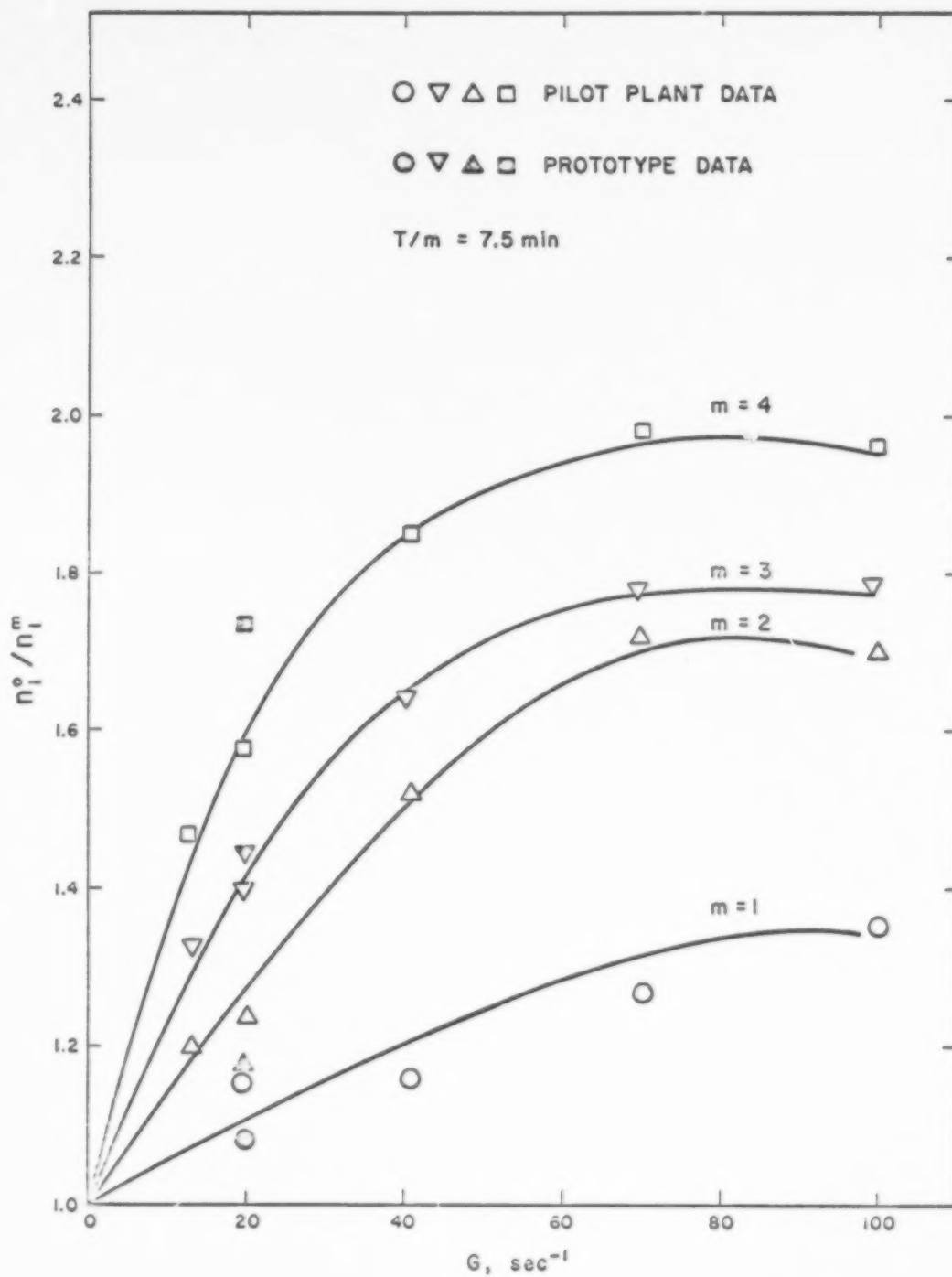


FIGURE 67. FLOCCULATION STUDY IN THE U.S.L. WATER TREATMENT PLANT

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